System Specification and Verification s
- Homework 2 -

Due on 28th of May 2026

1. Let fix a set $\mathcal{AP}$ of atomic propositions and let $\Sigma = 2^{\mathcal{AP}}$. The class of *eventuality* formulae is defined by the syntax

   $$\alpha := F\varphi \mid \alpha \vee \alpha \mid \alpha \wedge \alpha \mid X\alpha \mid \alpha U \alpha \mid \alpha R \alpha$$

   where φ ranges over all LTL formulae.

   Show that for all eventuality formulae α, for all $w \in \Sigma^\omega$ and all $0 \leq i \leq j$ we have

   $$w, j \models \alpha \implies w, i \models \alpha$$

2. Fix $\mathcal{AP} = \{p, q, r\}$. The aim is to see wether the CTL^* formula

   $$\varphi_1 = E((pUq)Ur)$$

   can be expressed in CTL. Consider the formula

   $$\varphi_2 = E((p \vee q)Ur)$$

   Use the definition of valid and equivalent formulae to prove that the formula $\varphi_1 \rightarrow \varphi_2$ is valid, but φ_1 and φ_2 are not equivalent.

3. Consider the game with the following winning condition for Player A:

   $$\Omega = V^* F_1 V^* F_2 ... V^* F_n V^\omega$$

   Hence, a play $\pi = v_0 v_1 v_2 ...$ is winning for Player A if it visits the sets F_1,...,F_n in the order, i.e., there exist indices $i_1 \leq i_2 \leq ... i_n$ such that $v_{i_j} \in F_j$ for all $1 \leq j \leq n$.

   (a) Give an algorithm to decide whether a vertex is winning for Player A. Include both an informal and formal description (intuitive description and the algorithm itself).

   (b) Argue that the algorithm is correct.

(c) What is the complexity of your algorithm?

(d) Is there always a positional winning strategy for Player A from a state in the winning region? If not, what is the needed memory for a winning strategy?

(e) Can you build a winning strategy for Player 0 from a given vertex?

4. **Bonus :** Consider the variation of the above game with the following winning condition for Player A:

$$\Omega = \bigcap_{i=1}^{n} V^* F_i V^\omega$$

How the algorithm from previous point changes? Answer at the same questions as above.