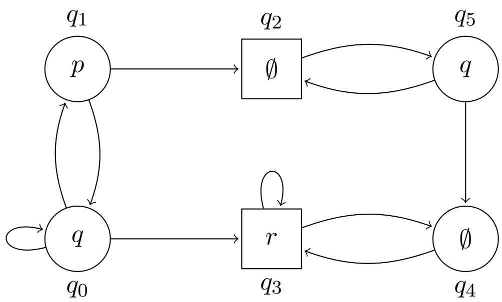
System Specification and Verification
- Seminar - Weeks 9,10 -

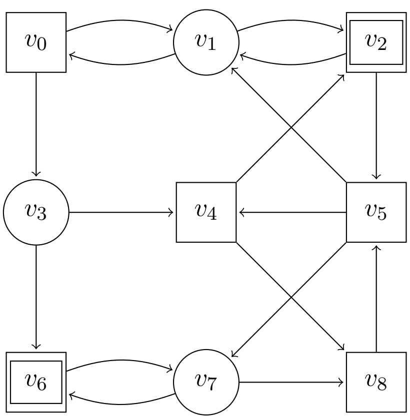
Spring 2026

1. Consider the game arena from below where Player A controls circle states and Player B controls square states.

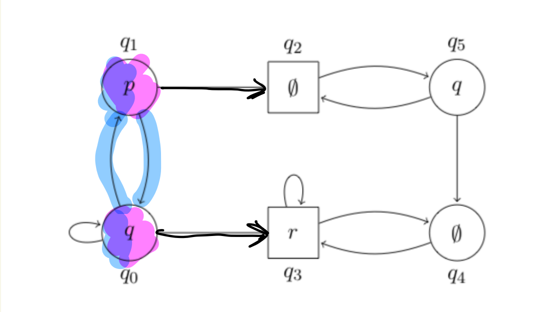


   Let consider the Buchi objective Büchi(T) for Player A, where $T = \{q_0, q_1, q_3\}$. Compute the states from which Player A has a winning strategy.

2. Give the states in the following game arena from which the player controlling squares has a strategy to visit finitely often the set $\{v_2, v_6\}$.



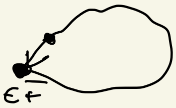
Ex 1



Player A → ○

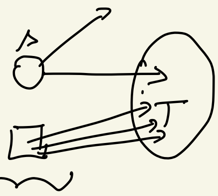
Player B → □

Obj A → Buchi$(\underbrace{\{q_0, q_1, q_3\}}_{T})$



Attractor în minimum 1 pas

$$\mathrm{Attr}^0_A(T) = \left\{ s \;\middle|\; \begin{array}{l} \text{dacă } s \in V_A \text{ a.î. } Succ(s) \cap T \neq \emptyset \\ \text{dacă } s \in V_B \text{ a.î. } Succ(s) \subseteq T \end{array} \right\}$$

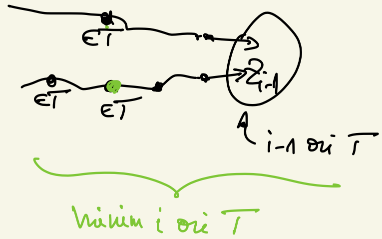


$$\mathrm{Attr}^i_A(T) = \mathrm{Attr}^{i-1}_A(T) \cup \ldots$$

$$Z_1 \supseteq Z_2 \supseteq Z_3 \ldots Z_\infty$$

$$Z_\infty = \bigcap_{i \geq 1} Z_i$$

$$Z_i = \begin{cases} T, & \text{dacă } i = 1 \\ \underbrace{\mathrm{Attr}_A(Z_{i-1})}_{\text{pct fix attr.}} \cap T, & \text{dacă } i > 1 \end{cases}$$



$$Win_A(G) = \mathrm{Attr}_A(Z_\infty)$$

$Z_1 = T = \{q_0, q_1, q_3\}$

$Z_2 = \mathrm{Attr}_A(Z_1) \cap T \;= \{q_0, q_1, q_4, q_5, q_2\} \cap \{q_0, q_1, q_3\} = \{q_0, q_1\}$

$\mathrm{Attr}^0_A(Z_1) = \{q_0, q_1, q_4\}$

$\mathrm{Attr}^1_A(Z_1) = \{q_0, q_1, q_4, q_5\}$

$\mathrm{Attr}^2_A(Z_1) = \{q_0, q_1, q_4, q_5, q_2\}$

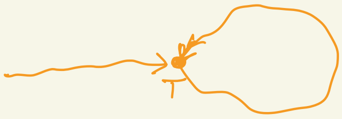
$\mathrm{Attr}^3_A(Z_1) = \{q_0, q_1, q_4, q_5, q_2\} = \mathrm{Attr}_A(Z_1)$

$Z_3 = \mathrm{Attr}_A(Z_2) \cap T \;= \{q_0, q_1\} \cap \{q_0, q_1, q_3\} = \{q_0, q_1\}$

$Attr_A^0(z_2) = \{q_0, q_1\}$

$Attr_A^1(z_2) = \{q_0, q_1\} = Attr_A(z_2)$

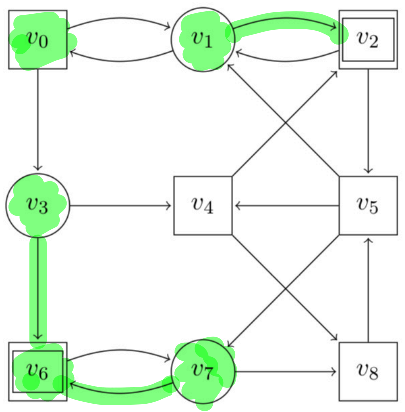
$z_2 = z_3 \Rightarrow Z_\infty = z_3 = \{q_0, q_1\}$



$Win_A(G) = Attr^A(z_\infty) = \{q_0, q_1\}$

2. Give the states in the following game arena from which the player controlling squares has a strategy to visit finitely often the set $\{v_2, v_6\}$.

B



Player B → □

$Obj_B = coBüchi(\underbrace{\{v_2, v_6\}}_{T})$

$\Downarrow$

Player A → O

$Obj_A = Büchi(\{v_2, v_6\})$

$Win_A(G)$ - to compute.

$Win_B(G) = V \setminus Win_A(G)$

Player A : Reach(R) - vizitează minim o stare din R (min o dată)

Player B : Safety($V \setminus R$) - nu vizitează nicio stare din R

$Win_B(G) = V \setminus Win_A(G)$

$Win_A(G) = Attr_A(\underline{z_\infty})$

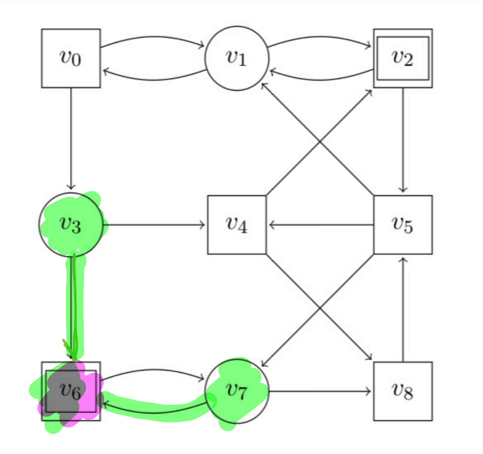
$z_1 = \{v_2, v_6\}$

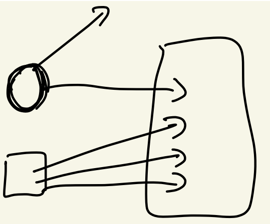
$z_2 = Attr_A(z_1) \cap T = \{v_1, v_3, v_7, v_0, v_6\} \cap \{v_2, v_6\} = \underline{\{v_6\}}$

$Attr_A^0(z_1) = \{v_1, v_3, v_7\}$

$Attr_A^1(z_1) = \{v_1, v_3, v_7, v_0, v_6\}$

$Attr_A^2(z_1) = \{v_1, v_3, v_7, v_0, v_6\} = Attr_A(z_1)$





$\underbrace{}_{Z_2}$

$Attr_A^{i-1}$

$$Z_3 = Attr_A(Z_2) \cap T$$

$$= Attr_A(v_6) \cap T = \{v_3, v_7, v_6\} \cap \{v_2, v_6\} = \underbrace{\{v_6\}}_{= Z_2}$$

$$Attr_A^0(Z_2) = \{v_3, v_7\} \leftarrow$$

$$Attr_A^1(Z_2) = \{v_3, v_7, v_6\}$$

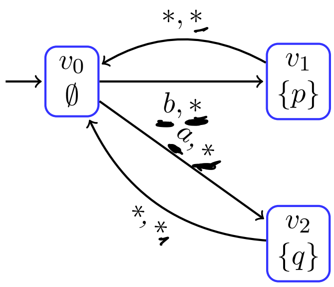
$$Attr_A^2(Z_2) = \{v_3, v_7, v_6\} = Attr_A(v_6)$$

$$Z_2 = Z_3 \Rightarrow Z_\infty = Z_3 = \{v_6\}$$

$$Win_A(G) = Attr_A(Z_\infty) = \{v_3, v_7, v_6\}$$

$$Win_B(G) = V \setminus Win_A(G) = \{v_0, v_1, v_2, v_4, v_5, v_8\}$$

3. Let consider the following arena.



$Act_0 = \{a, b\}$

$Act_1 = \{x, y\}$

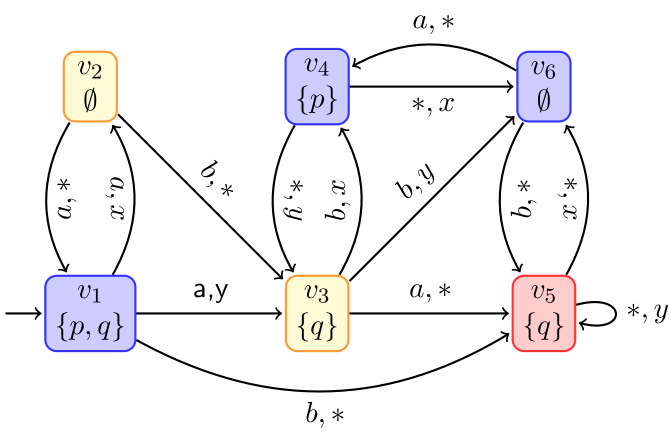
The aim is to enforce the infinite appearence of both p and q along the executions.

(a) Formalize as an LTL formula the above requirement.

(b) Is it true that all possible executions satisfy the specification?

(c) Give a strategy for Player A to enforce the satisfaction of the specification. Does this strategy need memory? Formalize it.

4. Consider the game arena from below where Player A has imperfect information and plays actions in $\{a, b\}$ and Player B is perfectly informed and plays actions in $\{x, y\}$. The indistinguishability relation for Player A is ilustrated with colors in the figure below (each color is an observation).

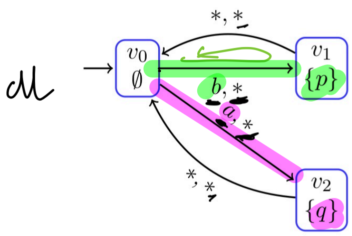


The aim is to give an observation-based strategy for Player A (if any) from the state v_1 to win the game $\mathcal{G}$ with the co-Büchi objective $\textsc{coBüchi}(\{v_5\})$.

a) Draw the perfect information game obtained by removing imperfect information (by computing possible states) from the game $\mathcal{G}$ with the initial state v_1. Don't forget to specify which is the winning condition in the obtained game.

b) Solve the obtained co-Büchi game.

3. Let consider the following arena.



$Act_0 = \{a, b\}$

$Act_1 = \{x, y\}$

The aim is to enforce the infinite appearence of both p and q along the executions.

(a) Formalize as an LTL formula the above requirement.

(b) Is it true that all possible executions satisfy the specification?

(c) Give a strategy for Player 0 to enforce the satisfaction of the specification. Does this strategy need memory? Formalize it.

a) $\varphi_1 = GF(p \vee q)$ X

$v_0\, v_1\, v_0\, v_1 \ldots$

$\varphi_2 = GF(p \wedge q)$ X — p și q simultan !

$\varphi_3 = (GF\, p) \wedge (GF\, q)$

b) Nu : $(v_0 v_1)^\omega \not\models \varphi_3$

MC pt LTL : $\mathcal{L}(\mathcal{A}_{\neg\varphi}) \cap \mathcal{L}(\mathcal{A}_{\mathcal{A}}) = \emptyset$ $\rightarrow$ NU – contraexemplu.

↳ $\neg\varphi \rightarrow$ NNF

↳ aut generalizat Büchi

↳ degeneralizarea

c) alternez între v_1 și v_2

memorie : ținem minte ultima stare vizitată dintre $\underset{p}{v_1}$ și $\underset{q}{v_2}$

(M_0, f_0, g_0)

stările de mem — actualizarea memoriei — dictează acțiunea

- starea inițială de memorie – m_1 / m_0

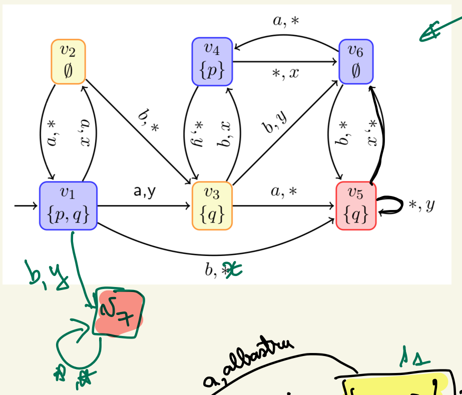
$M_0 = \{\underbrace{m_1}_{\text{am văzut } v_1}, \underbrace{m_2}_{\text{am văzut } v_2}, m_0\}$

$f_0 : V \times M_0 \rightarrow M_0$

$$f_0(v, m) = \begin{cases} m, & v = v_0 \\ m_1, & v = v_1 \\ m_2, & v = v_2 \end{cases}$$

$$g_0 : V \times V_0 \to Act_0 \qquad g_0(v,m) = \begin{cases} a, & \text{dacă } v \in \{v_1, v_2\} \\ a, & \text{dacă } v = v_0, (m = m_1 \text{ sau } m = m_0) \\ b, & \text{dacă } v = v_0, m = m_2 \end{cases}$$

Ex 4

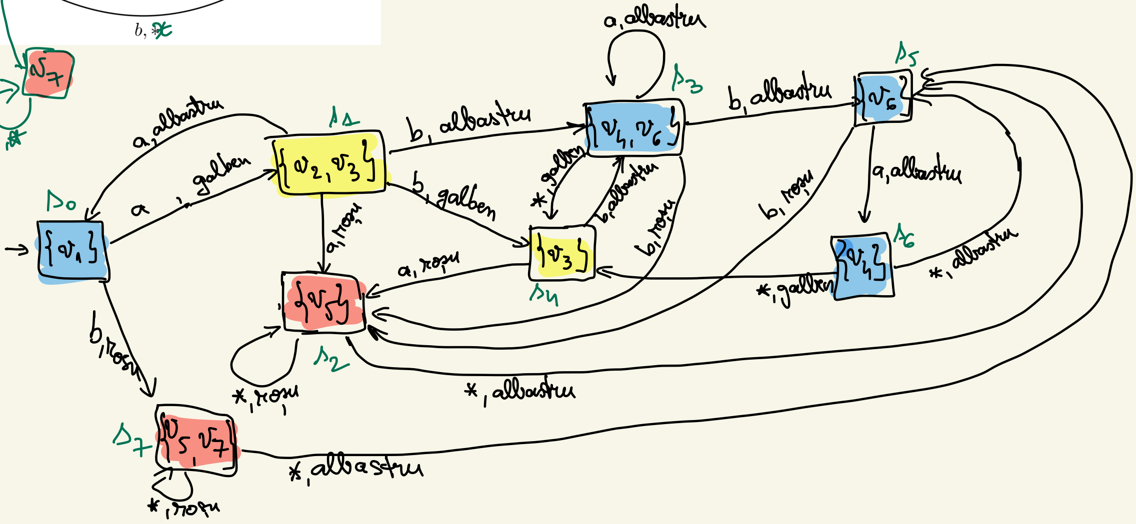


obj A : coBuchi $((v_5))^T$

$\{v_2, v_3\} \xrightarrow{b} \{v_3, v_4, v_6\}$

$v_4, v_6 \xrightarrow{a} \{v_4, v_6, v_3\}$

G' :



- perfect info

- Player A : $Act_A = \{a, b\}$

Player B : $Act_B = \{$albastru, roșu, galben$\}$

Obj A : - vizit de nr finit de ori mulțimi ce conțin stări din T (initial) $\{v_5\}$

↳ coBuchi$(\{s_2, s_7\})$

- Protagonist Player B : Obj Büchi$(\underbrace{\{s_2, s_7\}}_{T'})$

$$Z_1 = \{s_2, s_7\}$$

$$Z_2 = Attr^B(Z_1) \cap T'$$

$$Attr^0_B(Z_1) = \{s_1, s_0, s_4, s_3, s_5, s_2\} = Attr^1_B(Z_1)$$

. . . .

$\mathcal{W}\mathrm{in}_B(G')$

$\mathcal{W}\mathrm{in}_A(G') = V \setminus \mathcal{W}\mathrm{in}_B(G')$