1. Build one non-deterministic Büchi word automaton for the following formulas:

- $\varphi_1 = \neg(GFb)$
- $\varphi_2 = G(p \to X(q\mathcal{U}r))$
- $\varphi_3 = (G(p \to q)) \to G\alpha$ where $\alpha = F(p \wedge Xp)$.
- $\varphi_4 = \neg(a\mathcal{U}X(a \wedge \neg b))$

$\mathcal{L}(\mathcal{A}_\varphi) = \mathcal{L}(\varphi)$

i Write the formula in negative normal form

ii Draw the reduction graph starting from φ.

iii Give the sets $Red(\{\varphi\})$ and $Red_\alpha(\{\varphi\})$.

iv Draw the transitions starting from state $\{\varphi\}$ in the GBA $\mathcal{B}_\varphi$.

v Complete the construction and draw the automaton $\mathcal{B}_\varphi$.
Indicate clearly the accepting conditions.

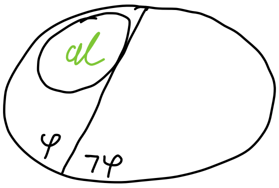
vi Compute the non-deterministic Büchi automaton $\mathcal{A}_\varphi$.

MC LTL

$\mathcal{M} \models \varphi$

$\mathcal{L}(\mathcal{M}) \subseteq \mathcal{L}(\varphi)$

$\mathcal{L}(\mathcal{M}) \cap \mathcal{L}(\neg\varphi) = \emptyset$

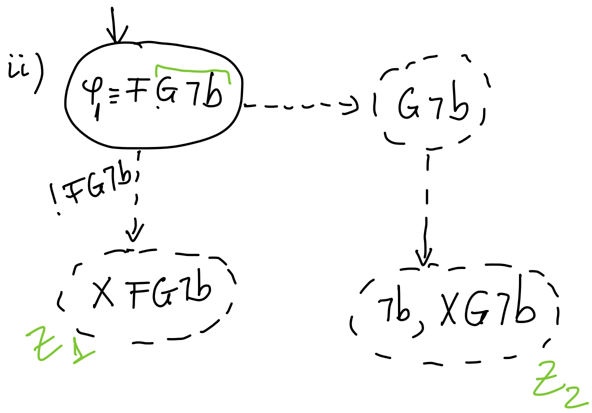


$\mathcal{L}(\mathcal{A}_\mathcal{M} \cap \mathcal{A}_{\neg\varphi}) = \emptyset$

i) $\varphi_1 = \neg(GFb) \equiv F\neg Fb \equiv FG\neg b$ – NNF

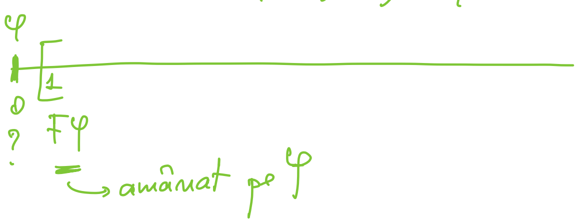
$\neg G\psi \equiv F\neg\psi$

ii)



$F\psi \in Y \dashrightarrow (Y \setminus \{F\psi\}) \cup \{\psi\}$

$\xrightarrow{!F\psi} (Y \setminus \{F\psi\}) \cup \{XF\psi\}$

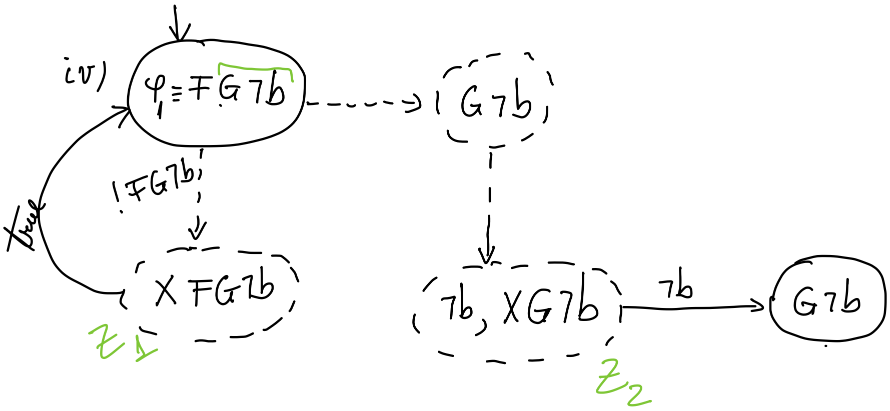


iii) $\alpha = FG\neg b$

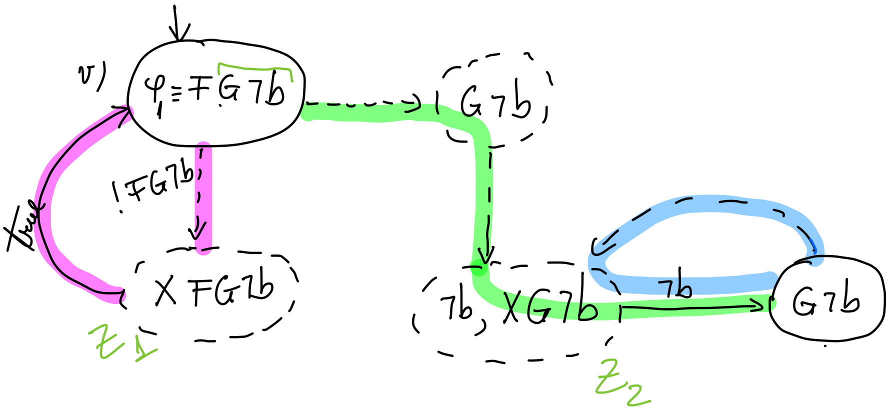
$Red(\varphi_1) = \{\{XFG\neg b\}, \{\neg b, XG\neg b\}\}$

$Red_\alpha(\varphi_1) = \{\{\neg b, XG\neg b\}\}$

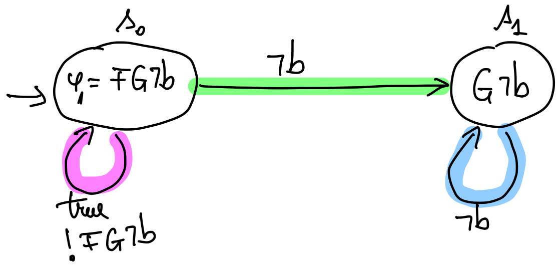
iv)



v)



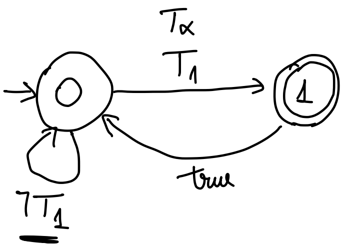
$\mathcal{B}_{\varphi_1}$:



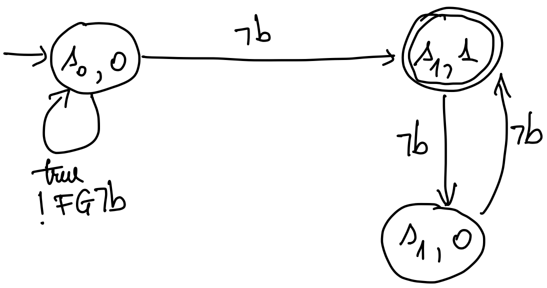
$T = \{ T_1, \cancel{T_2, \ldots} \}$

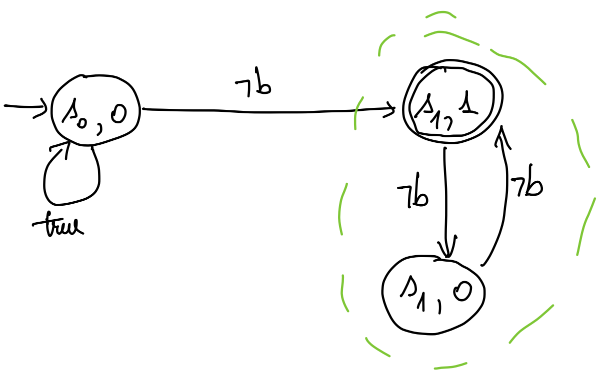
$T_\alpha = \{ s_0 \to s_1,\ s_1 \to s_1 \}$

$\mathcal{D}_1$:

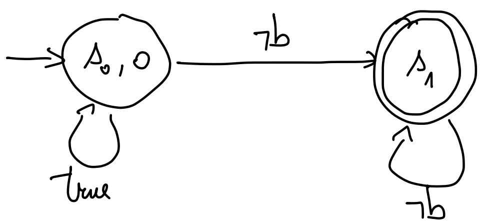


$\mathcal{B}_{\varphi_1} \otimes \mathcal{D}_1$:

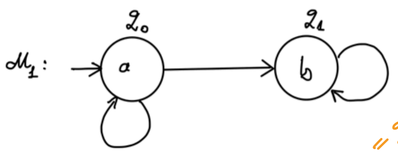




$\mathcal{A}_{\varphi_1}$:



2. Verify if the below transition system satisfies GFb.



$\mathcal{M}_1 \models \overbrace{GFb}^{\varphi}$

$trace(\pi_1) = aaaaa\ldots$ ($q_0 q_0 q_0 q_0 \ldots$)

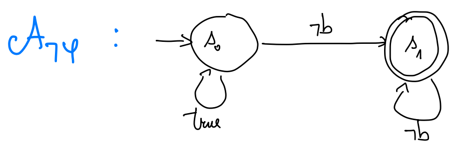
$trace(\pi_2) = a^* b^\omega$ ($(q_0)^* q_1^\omega$)

$$\mathcal{L}(\mathcal{A}_\mathcal{M} \cap \underbrace{\mathcal{A}_{\neg\varphi}}) = \emptyset$$

↳ $\mathcal{A}_{\neg GFb}$

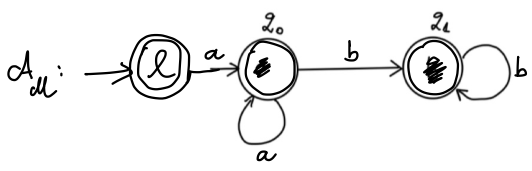
$Traces(\mathcal{M}) = \{a^\omega, a^* b^\omega\}$

$\neg\varphi = \neg GFb$ → construim automatul nedet. Büchi pt $\neg\varphi = \neg GFb$



$\underline{Traces(\mathcal{M})} \subseteq \mathcal{L}(\varphi)$ ($\varphi = GFb$) NU

$\mathcal{L}(\mathcal{A}_\mathcal{M}) \subseteq \mathcal{L}(\varphi)$



- adaug starea ℓ "înaintea" stărilor inițiale
- toate stările sunt finale
- toate tranzițiile către starea q_i "citesc" litera $L(q_i)$

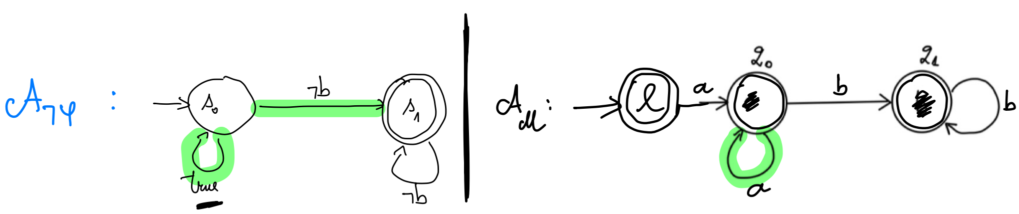
$$\mathcal{L}(\mathcal{A}_\mathcal{M}) \cap \mathcal{L}(\neg\varphi) \stackrel{?}{=} \emptyset$$

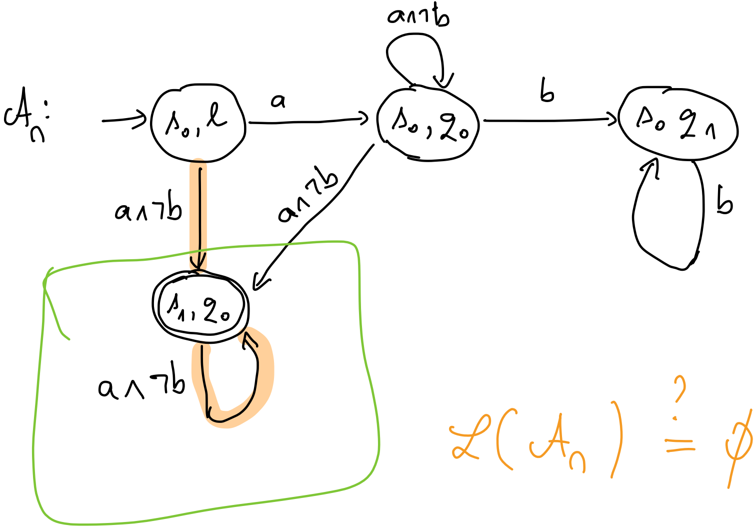
$$\mathcal{L}(\mathcal{A}_\mathcal{M} \cap \mathcal{A}_{\neg\varphi}) \stackrel{?}{=} \emptyset$$

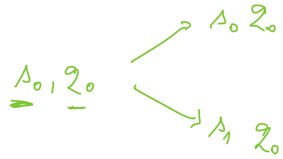
└ toate stările finale ⇒ caz. particular

$AP = \{a, b\}$

tranziție cu toate combinațiile de val de adevăr







$\mathcal{L}(A_\cap) \overset{?}{=} \emptyset$ NU $\longrightarrow$ contraexemplu

$w = (a \wedge \neg b)\,(a \wedge \neg b)^\omega$

$s_0 l \longrightarrow s_1 q_0$ ↻

componentă tare conexă ce conține stare finală și este reachable din starea inițială

$w = a^\omega$