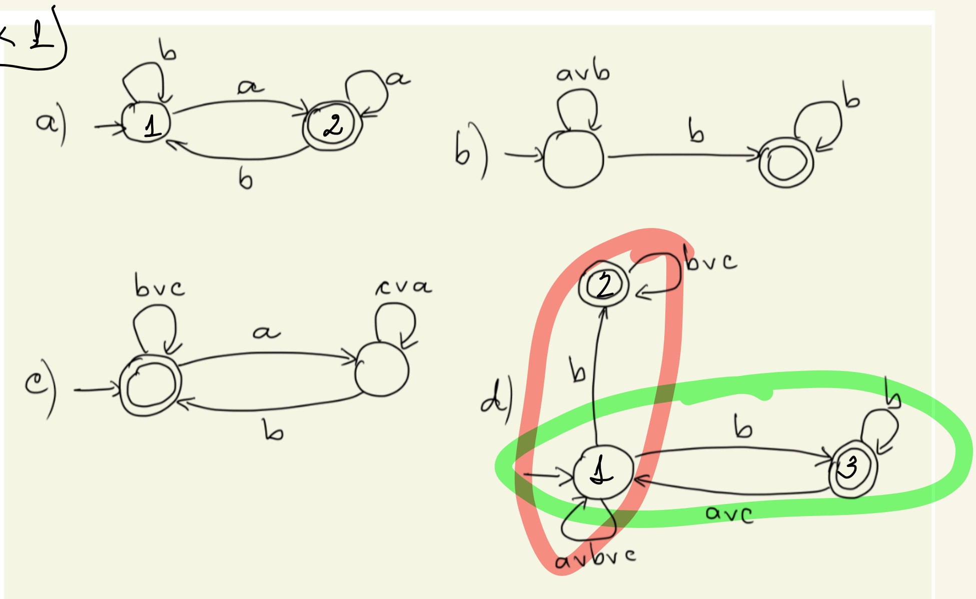
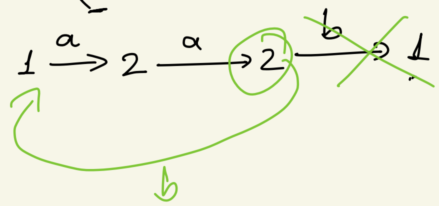
Ex 1)



$w = (aab)^\omega$



- Give the set of atomic propositions used.
- What is the language of each automaton?
- Give the LTL formula such that the language of the automaton

a) $AP = \{a, b\}$

$T = \{2\}$

$inf(\rho) \cap T \neq \emptyset$

$\mathcal{L} = \{ \, b^*(a^+ b^*)^\omega \, \}$

$(b^* a)^\omega$

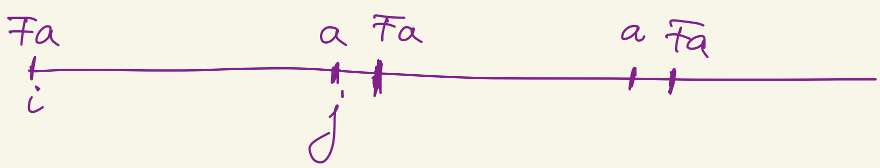
$\cancel{b^* a^\omega}$

$w = (ab)^\omega$

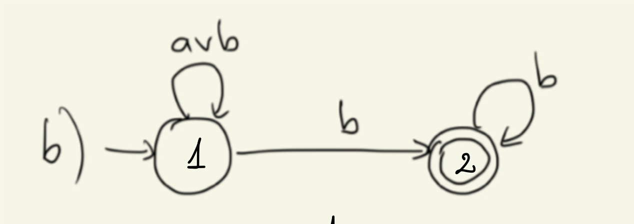
ababaaaa a

$\varphi = GFa$

$w, i \models \underbrace{Fa}_{\top \mathcal{U} a}$ iff there is $j \geq i$ s.t. $w, j \models a$



b)



$\mathcal{A}_2$

$AP = \{a, b\}$

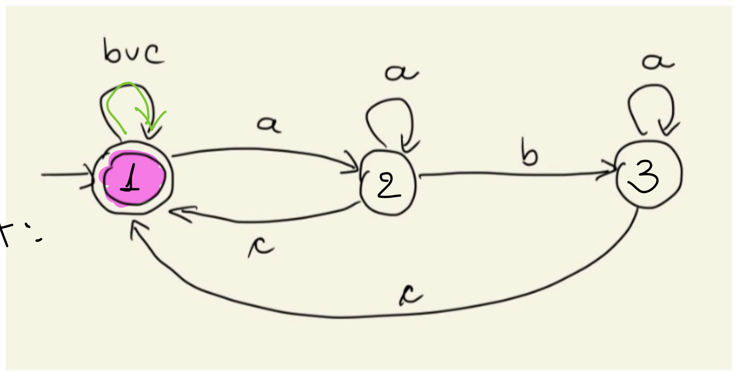
$\mathcal{L}(\mathcal{A}_2) = \{(a+b)^* b^\omega\}$ $\qquad (a^* b^*)^* b^\omega$

$w_1 = aba\underbrace{bbbb\ldots}_{b^\omega}$

$\varphi = FGb$

$\boxed{Ex2}$

$\mathcal{A}$:



(a) $w_1 = (bac)^\omega$ ✓ $\qquad \rho_1 = 1 \xrightarrow{b} 1 \xrightarrow{a} 2 \xrightarrow{c} 1$

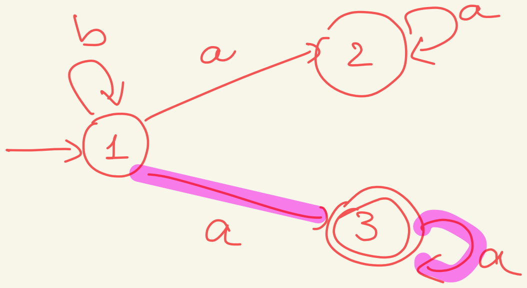
(b) $w_2 = aab(ac)^\omega$ ✓

(c) $w_3 = ba(abab)^\omega$

(d) $w_4 = a^\omega$ $\qquad a^+ a^\omega$

(e) $w_5 = (abc)^\omega$

b) $\rho_2 = 1 \xrightarrow{a} 2 \xrightarrow{a} 2 \xrightarrow{b} 3 \xrightarrow{a} 3 \xrightarrow{c} 1 \xrightarrow{a} 2 \xrightarrow{c} 1$



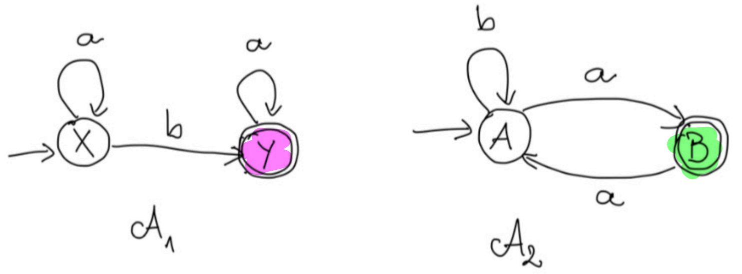
$b^* a^\omega$

$(s_1, a, s_2) \in Q \times 2^{AP} \times Q$

d) $\rho_4 = 1 \xrightarrow{a} 2 \circlearrowleft a$ ⟵ singura execuție posibilă!

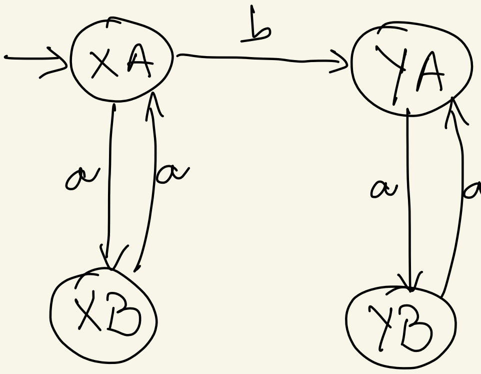
(nu acceptă w_4 $\Rightarrow$ $w_4 \notin \mathcal{L}(\mathcal{A})$)

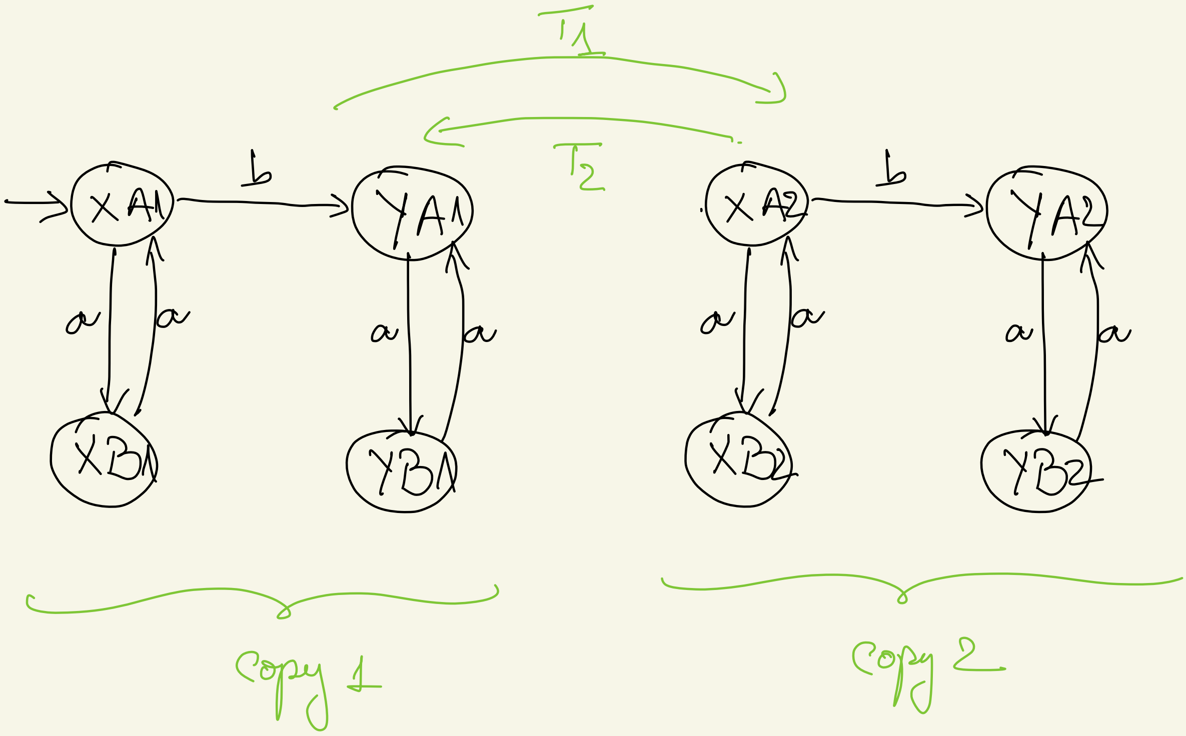
Ex 3

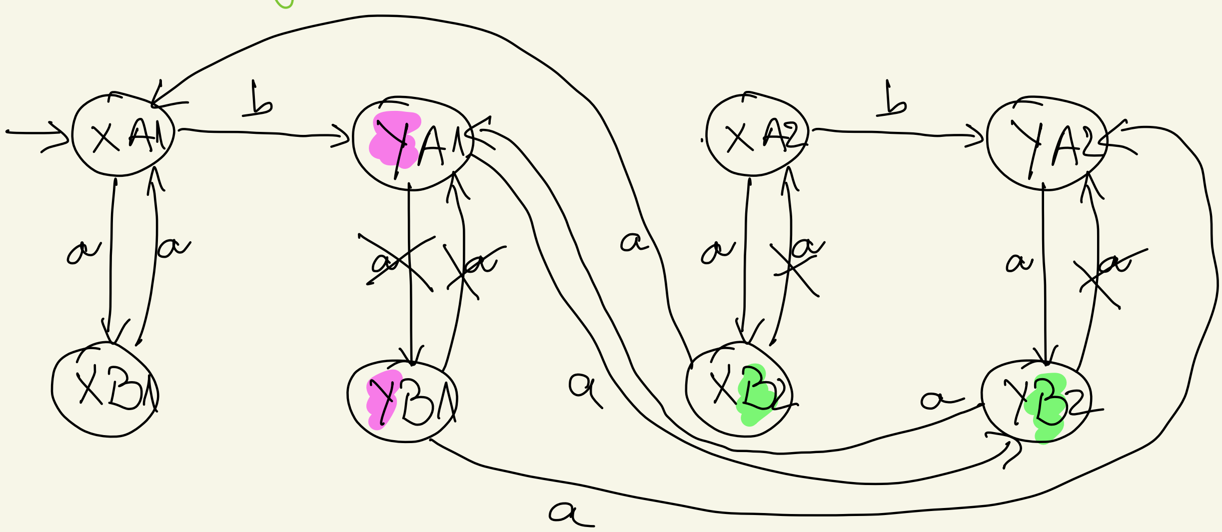


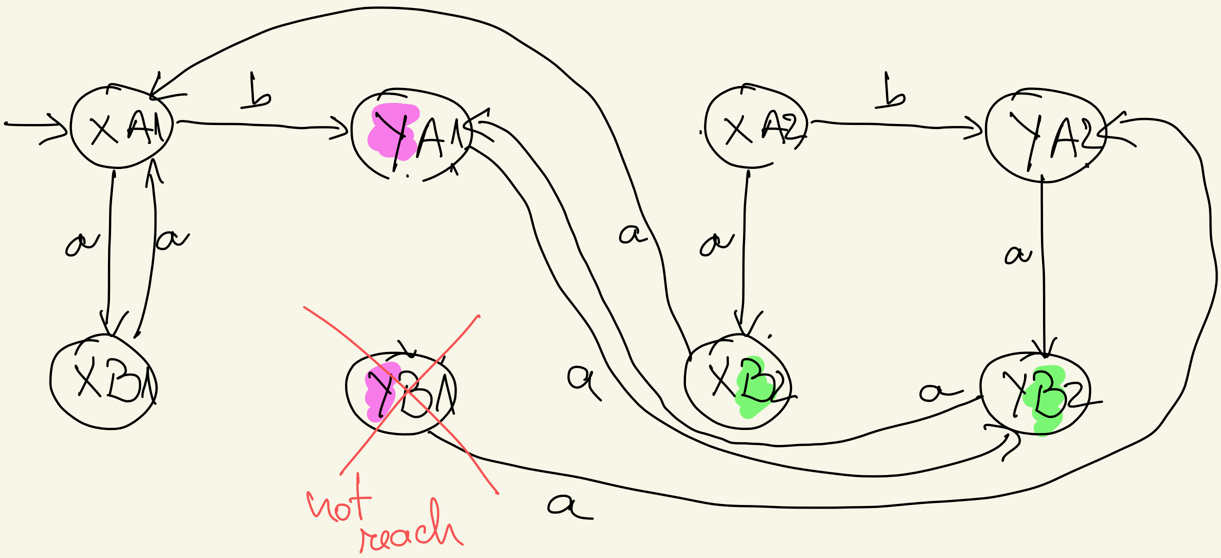
Build the automaton $A_\cap$ s.t. $\mathcal{L}(A_\cap) = \mathcal{L}(A_1) \cap \mathcal{L}(A_2)$

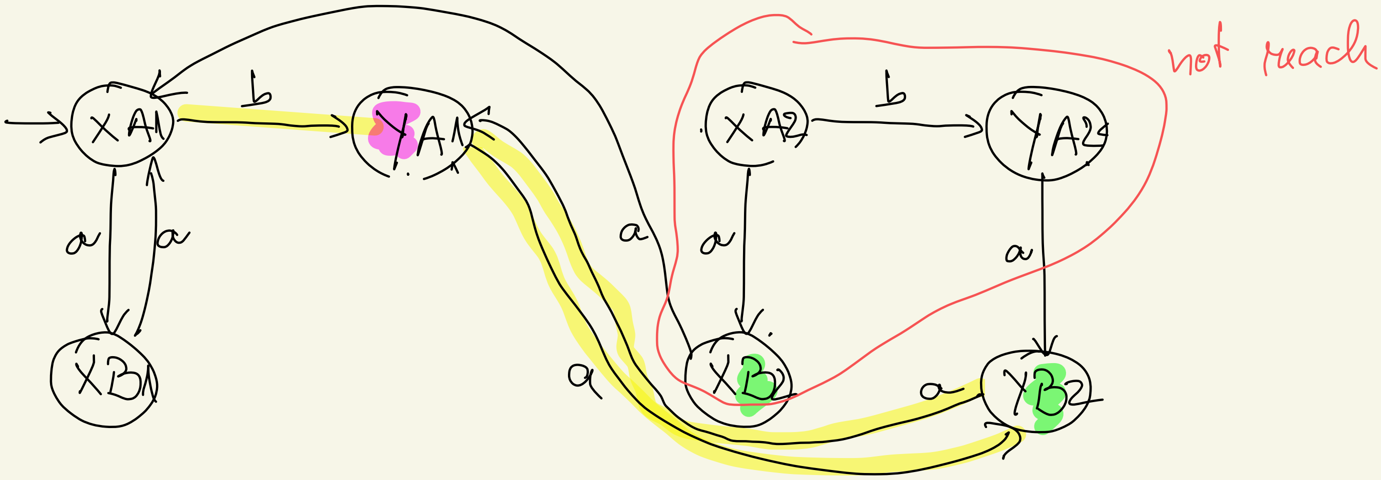
– cazul general

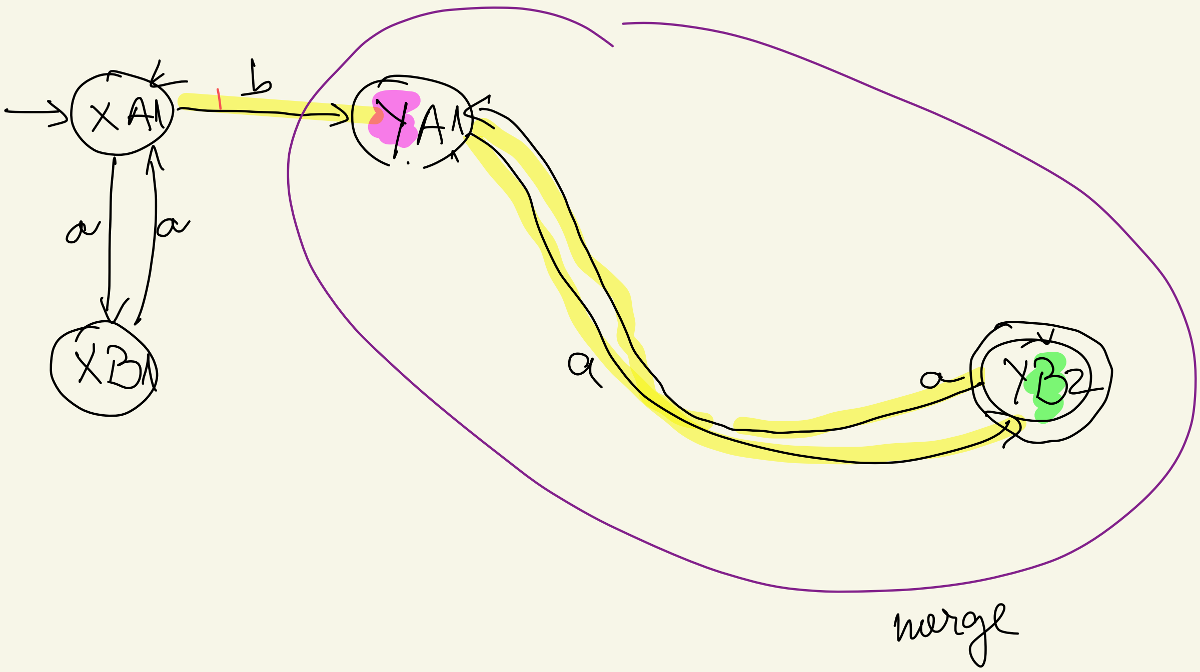


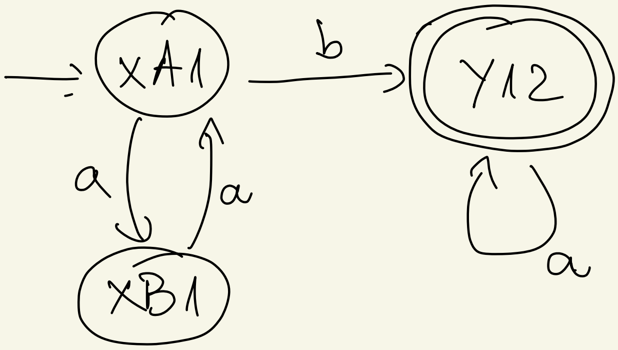





XA1
YA1
XB1
YB1
XA2
YA2
XB2
YB2
a
b
not reach


XA1
YA1
XB1
XA2
YA2
XB2
YB2
a
b
not reach


XA1
YA1
XB1
YB2
a
b
merge



$\mathcal{L} = \{(aa)^* b\, a^\omega\}$

$Q \times 2^{AP} \times Q$

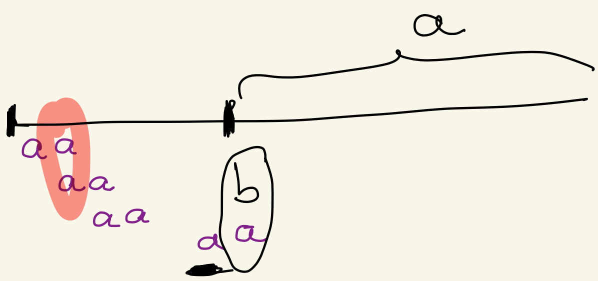
$w = \{a\}\{ab\}\{b\}$

aba

$\varphi = G(b \to X\, G(a \wedge \neg b))$

$\wedge\, F b$

$\wedge\, G((a \wedge X \neg b) \to X a)$



$(a \wedge X a)\, \mathcal{U}\, b$

aaaba

4. (Ultimately Periodic Words) An *ultimately periodic word* over 2^{AP} is a word of the form $u \cdot v^\omega$ with $u \in (2^{AP})^*$ and $v \in (2^{AP})^+$.

   Prove that any nonempty recognizable language by a Büchi automaton contains an ultimately periodic word.

Let $L \neq \emptyset$ be arbitrar, recognizable by a Büchi automaton.

Prove that there is $w = u \cdot v^\omega \in L$

$u \in (2^{AP})^*$ $\quad v \in (2^{AP})^+$

L - recognizable by a Büchi automaton $\Rightarrow$.

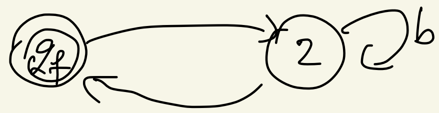
$\Rightarrow \exists\ \mathcal{A} = (Q, 2^{AP}, Q_0, \delta, T)$ a Büchi automaton s.t. $\mathcal{L}(\mathcal{A}) = L \neq \emptyset$

$\Rightarrow \exists\ w' = w_0 w_1 w_2 \dots \in \mathcal{L}(\mathcal{A})$ (accepted by $\mathcal{A}$)

$\Rightarrow \exists\ \rho' = q_0 q_1 q_2 \dots$ a path on w' s.t.

- $q_0 \in Q_0$ and
- for each $i \geq 0$, $(q_i, w_i, q_{i+1}) \in \delta$
- there is $q_f \in T$ s.t. $q_i = q_f$ for infinitely many $i \geq 0$.

$$\rho' = q_0 \xrightarrow{w_0} q_1 \xrightarrow{w_1} q_2 \dots q_{j-1} \xrightarrow{w_{j-1}} \underset{\substack{= \\ \text{first } q_f}}{q_j} \xrightarrow{w_j} \underbrace{q_{j+1} \xrightarrow{w_{j+1}} \dots \xrightarrow{w_{k-1}}}\ \underset{k}{q_f} \underbrace{\dots\dots} q_f \dots$$



Let $j \geq 0$ be the first s.t. $q_j = q_f$.

q_f - visited infinitely often $(q_f \in \inf(\rho') \cap T)$ $\Rightarrow$ there is a finite path (the green one from above in ρ') to go back to q_f once we leave

Let u be the prefix up to position j in w'

$$u = w_0 w_1 \dots w_{j-1}$$

Let k be the second occ. of q_f and

$$v = w_j w_{j+1} \dots w_{k-1}$$

$$w = u(v)^\omega$$

$$\rho = \underbrace{q_0 \cdots\cdots q_f}_{u} \cdots\cdots \quad v$$