1. Let consider a boolean circuit with input x, output y and two registers r_1 and r_2.

   Translate the following properties as LTL formulas over $\mathcal{AP} = \{x, y, r_1, r_2\}$:

   (a) "it is impossible to get two consecutive 1 as output"

   (b) "each time the input is 1, at most two ticks later, the output will be 1"

   (c) "each time the input is 1, the register contents remain the same over the next tick."

   (d) "register r_1 is infinitely often 1"

   (e) "register r_1 has a finite number of times value 1"

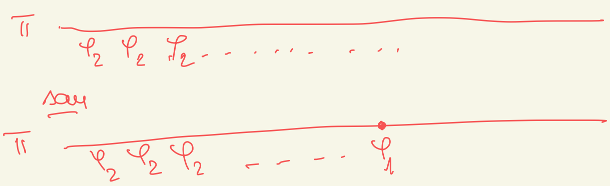
   (f) "register r_1 has value 1 exactly one time".

LTL : $\varphi ::= p \mid \neg\varphi \mid \varphi \vee \varphi \mid X\varphi \mid \varphi\,\mathcal{U}\,\varphi$

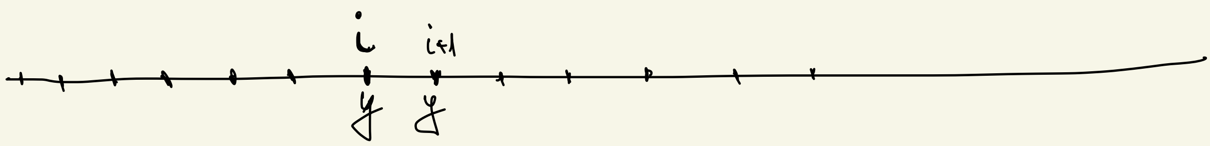
$F\varphi \equiv \top\,\mathcal{U}\,\varphi$

$G\varphi \equiv \neg F \neg \varphi$

$\varphi_1 \mathcal{R} \varphi_2 \equiv (G\varphi_2) \vee (\varphi_2\,\mathcal{U}\,\varphi_1)$



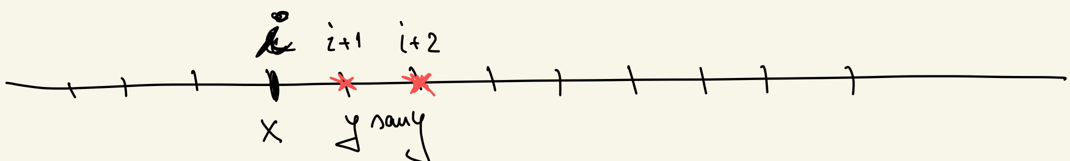
a)



$\pi, i \models G\neg(\underbrace{y \wedge Xy}_{\text{2 valori de 1 consecutive pt output}})$

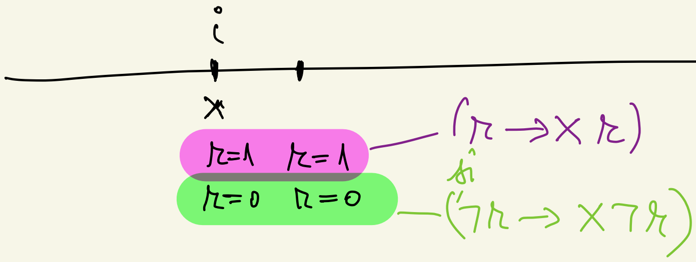
$\neg F(y \wedge Xy)$

b)



$\varphi = G(x \to (Xy \vee XXy))$

c)

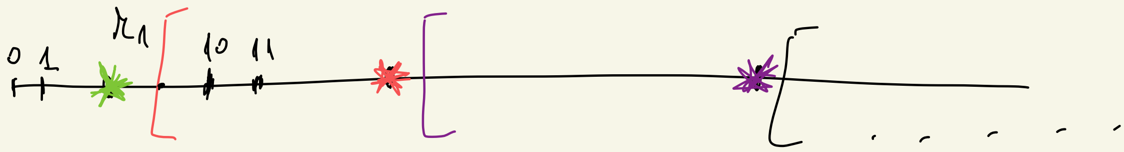


$$(r \to Xr) \wedge (\neg r \to \neg X r)$$
$$(r \to Xr) \wedge (\neg\neg r \vee \neg X r)$$
$$(r \to Xr) \wedge (\neg X r \vee r)$$
$$(r \to Xr) \wedge (X r \to r)$$
$$\equiv \underbrace{r \leftrightarrow X r}_{}$$

r păstrează val la momentul următor

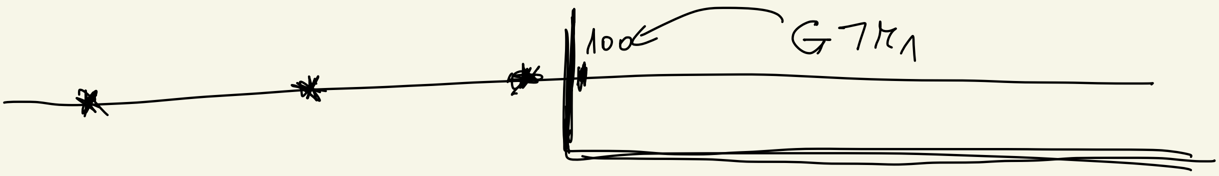
$$\varphi = G\big(x \to ((r_1 \leftrightarrow X r_1) \wedge (r_2 \leftrightarrow X r_2))\big)$$

d)



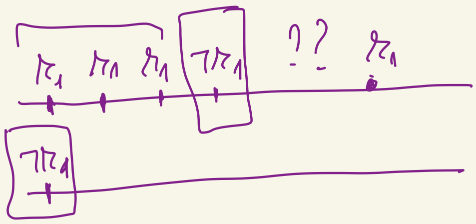
$$\varphi = G F r_1$$

e)



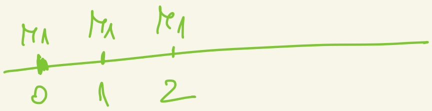
$$\cancel{\varphi = F(r_1 \mathcal{U} \neg r_1) \equiv F F \neg r_1 \equiv F \neg r_1}$$

$$\underbrace{r_1 \mathcal{U} \neg r_1}_{\equiv F \neg r_1}$$

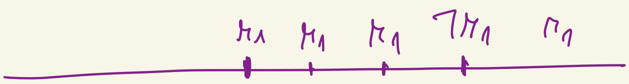


$$\cancel{\varphi_1 = r_1 \to X \neg r_1}$$

enunț permite:



$$\cancel{\varphi_2 = F(\neg r_1 \mathcal{R} r_1)}$$

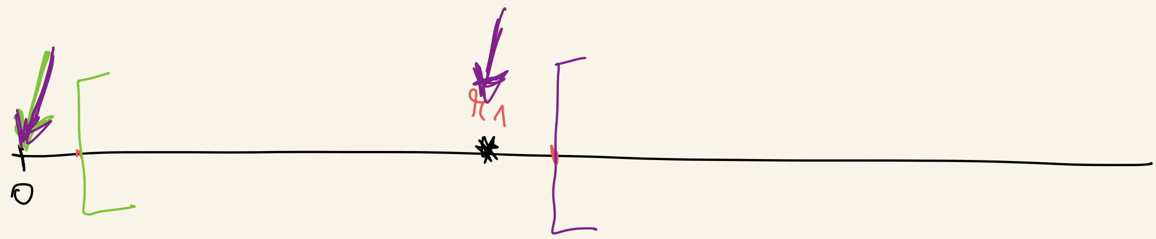


$\varphi_3 = \cancel{(F \neg r_1)\ \mathcal{R}\ r_1}$

$\varphi_4 = \cancel{(F r_1)}\ \mathcal{U}\ (F \neg r_1)$

$\varphi = F G (\neg r_1)$

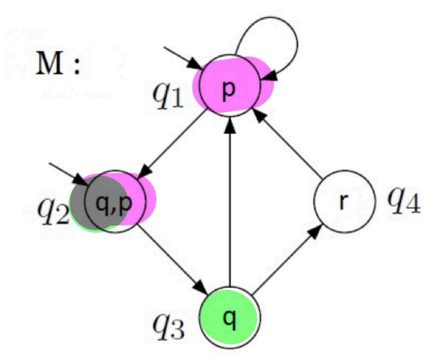
f)

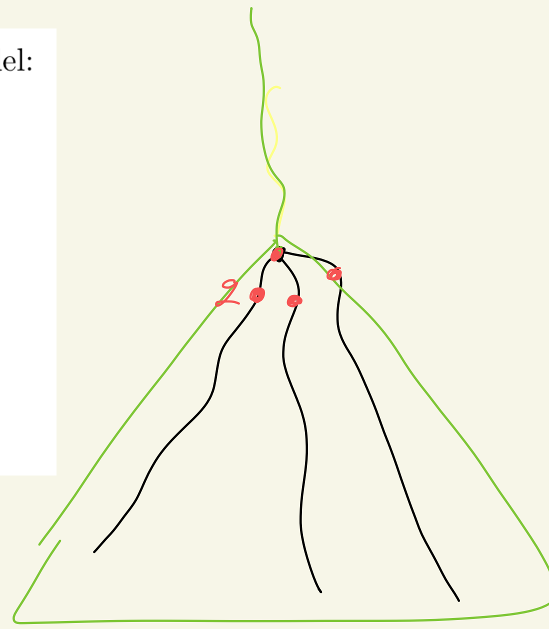


$\varphi = \underbrace{F r_1}_{\text{măcar o apariție}} \wedge G(r_1 \to X G \neg r_1)$

acum dacă r_1 atunci de la pasul următor r_1 nu mai apare

2. Compute $[\![AXq]\!]$, $[\![EXp]\!]$ and $[\![E(q\mathcal{U}r)]\!]$ in the following model:





$AXq \equiv \neg EX \neg q$

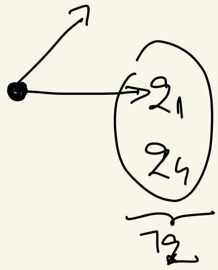
$[\![AXq]\!] = [\![\neg EX\neg q]\!] = S \setminus [\![EX \neg q]\!] =$

$= S \setminus pre_\exists([\![\neg q]\!]) = S \setminus pre_\exists(S \setminus [\![q]\!]) =$

$= S \setminus pre_\exists(S \setminus \{q_2, q_3\})$

$= S \setminus pre_{\exists}(\{q_1, q_4\})$

$= S \setminus \underbrace{\{q_1, q_4, q_3\}}_{\models EX\neg q}$



$= \{q_2\}$

$[\![AXq]\!] = pre_{\forall}([\![q]\!]) = pre_{\forall}(\{q_2, q_3\}) = \{q_2\}$

$[\![EXp]\!] = pre_{\exists}([\![p]\!]) = pre_{\exists}(\{q_1, q_2\}) = \{q_1, q_3, q_4\}$

$[\![E(q\,U\,r)]\!] =$

$[\![q]\!] = \{q_2, q_3\}$

$Y = \emptyset \quad Z = [\![r]\!] = \{q_4\}$

$[\![r]\!] = \{q_4\}$

- $MC^{EU}_{CTL}(\varphi_1, \varphi_2)$ is computed as:
  - $Y := \emptyset$; $Z := [\![\varphi_2]\!]$;
  - **while** $Z \not\subseteq Y$ **do**:
    - $Y = Y \cup Z$;
    - $Z = pre_{\exists}(Y) \cap [\![\varphi_1]\!]$
  - **return** Y

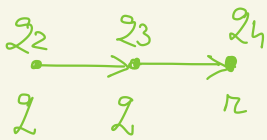
Stop when cannot add nodes in Y!

$\cdot\; Z \not\subseteq Y$

$Y = Y \cup Z = \{q_4\}$

$Z = \{q_3\} \cap [\![q]\!] =$

$= \{q_3\} \cap \{q_2, q_3\} = \{q_3\}$



$\cdot\; Z \not\subseteq Y$

$Y = Y \cup Z = \{q_4, q_3\}$

$Z = \{q_3, q_2\} \cap \underbrace{\{q_2, q_3\}}_{[\![q]\!]} = \{q_2, q_3\}$

$\cdot\; Z \not\subseteq Y$

$Y = \{q_4, q_3, q_2\}$

$Z = \{q_3, q_2, q_1\} \cap \{q_2, q_3\} = \{q_2, q_3\}$

$\cdot\; Z \not\subseteq Y$ "F".

Deci $[\![E(q \,\mathcal{U}\, r)]\!] = \{q_4, q_3, q_2\}$

3. (CTL^*) Are the following formulas equivalent?

(a) $AXAG\varphi$ and $AXG\varphi$

$AXAG\varphi \equiv AXG\varphi$ ddacă pt orice model $\mathcal{M}$, pt orice execuție π în $\mathcal{M}$ și pt orice poziție $i \geq 0$

avem $\pi, i \models AXAG\varphi$ ddacă $\pi, i \models AXG\varphi$

Fie $\mathcal{M}$ arbitrar și π în $\mathcal{M}$ fixat arbitrar și $i \geq 0$ fixat aleator

$\pi = s_0\, s_1\, s_2\, s_3 \ldots s_i \ldots$ s_0' $s_1' - s_2'$

$\pi, i \models AXAG\varphi$ ddacă pt orice $\pi' = s_0'\, s_1'\, s_2' \ldots$ a.î. $s_i = s_0'$

avem $\pi', 0 \models XAG\varphi$

ddacă pt orice $\pi' = s_0'\, s_1'\, s_2' \ldots$ a.î. $s_i = s_0'$

avem $\pi', 1 \models AG\varphi$

ddacă pt orice $\pi' = s_0'\, s_1'\, s_2' \ldots$ a.î. $s_i = s_0'$ ($s_1' = s_0''$)

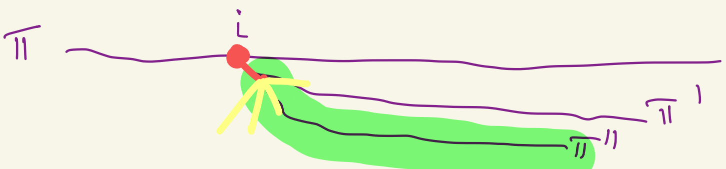
avem pt orice $\pi'' = s_0''\, s_1''\, s_2'' \ldots$ a.î. $s_1' = s_0''$

avem $\pi'', 0 \models G\varphi$

ddacă pt orice $\pi' = s_0'\, s_1'\, s_2' \ldots$ a.î. $s_i = s_0'$ ($s_1' = s_0''$)

avem pt orice $\pi'' = s_0''\, s_1''\, s_2'' \ldots$ a.î. $s_1' = s_0''$

avem pt orice $j \geq 0$ avem $\pi'', j \models \varphi$



$\pi, i \models \underline{A} X G \varphi$ ddacă pt orice $\pi' = s'_0 s'_1 s'_2 \dots$ a.î. $s_i = s'_0$

avem $\pi', 0 \models X G \varphi$

ddacă pt orice $\pi' = s'_0 s'_1 s'_2 \dots$ a.î. $s_i = s'_0$

avem $\pi', 1 \models G \varphi$

ddacă pt orice $\pi' = s'_0 s'_1 s'_2 \dots$ a.î. $s_i = s'_0$

avem pt orice $j \geq 1$ avem $\pi', j \models \varphi$