System Specification and Verification
- Seminar - Week 2 -

Spring 2026

1. Let $\mathcal{M} = \langle W, R, L \rangle$ and $\mathcal{M}' = \langle W', R', L' \rangle$ be two Kripke structures. A relation $Z \subseteq W \times W'$ is said to be *bisimulation* between $\mathcal{M}$ and $\mathcal{M}'$ if the following conditions are satisfied:

   (a) For all $w \in W$ and $w' \in W'$ such that $(w, w') \in Z$ and each $p \in \mathcal{AP}$, it holds $p \in L(w)$ if and only if $p \in L'(w')$;

   (b) If $(w, w') \in Z$ and $(w, v) \in R$, then there exists $v' \in W'$ such that $(v, v') \in Z$ and $(w', v') \in R'$ (*forth condition*);

   (c) If $(w, w') \in Z$ and $(w', v') \in R$, then there exists $v \in W$ such that $(v, v') \in Z$ and $(w, v) \in R$ (*back condition*);

   Worlds w and w' are called bisimilar, $w \leftrightsquigarrow_{\mathcal{M},\mathcal{M}'} w'$, if there is a bisimulation Z between $\mathcal{M}$ and $\mathcal{M}'$ with $(w, w') \in Z$.

   An example of bisimulation is in Figure 1.

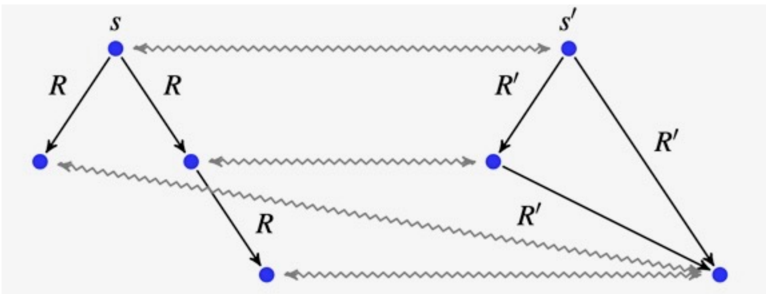

Figure 1: Example of bisimulation

Prove the following Proposition:

Proposition 1 *Let $\mathcal{M} = \langle W, R, L \rangle$ and $\mathcal{M}' = \langle W', R', L' \rangle$ be two Kripke structures. Let $w \in W$ and $w' \in W'$ such that $w \leftrightsquigarrow_{\mathcal{M},\mathcal{M}'} w'$. Then w and w' are modally equivalent, i.e., for each formula Modal Logic φ,*

$$\mathcal{M}, w \models \varphi \textit{ iff } \mathcal{M}', w' \models \varphi$$

2. Prove that

   Theorem 1 *Let $\mathcal{F} = \langle W, R \rangle$ be a frame. The following statements are equivalent:*

   - *R is reflexive;*
   - *$\Box\varphi \to \varphi$ is valid in $\mathcal{F}$, for any basic modal logic formula φ;*

3. Prove that

   Theorem 2 *Let $F = (W, R)$ be a frame. The following statements are equivalent:*

   - *R is transitive;*
   - *$\Box\varphi \to \Box\Box\varphi$ is valid in $\mathcal{F}$, for any basic modal logic formula φ;*