System Specification and Verification
- Seminar - Week 1 -

Spring 2026

1. Consider the following model: $\mathcal{M} = \langle W, R, L\rangle$ where: $W = \{a, b, c, d, e\}$, $R = \{(a, b), (a, e), (b, c), (b, e), (d, d), (e, e)\}$ and $L(a) = \{p, q\}$, $L(b) = \{p\}$, $L(c) = \{p, q\}$, $L(d) = \{q\}$, $L(e) = \emptyset$.

   Determine if:

   (a) $a \models p$
   (b) $a \models \Box\neg q$
   (c) $a \models \Box\Box q$
   (d) $a \models \Diamond p$
   (e) $a \models \Box\Diamond\neg q$
   (f) $a \models \Diamond\Diamond(p \wedge q) \wedge \Diamond\top$

2. For the model described at Exercise 1, find a world that satisfy and a world that does not satisfy the following formulas:

   (a) $\Box\neg p \wedge \Box\Box\neg p$,
   (b) $\Diamond q \wedge \neg\Box q$
   (c) $\Diamond p \vee \Diamond q$
   (d) $\Diamond(p \vee \Diamond q)$
   (e) $\Box p \vee \Box\neg p$
   (f) $\Box(p \vee \neg p)$

3. For each pair of formulas, find a Kripke model that satisfy only one of them:

   (a) $\Box p$ and $\Box\Box p$
   (b) $\Box\neg p$ and $\neg\Diamond p$
   (c) $\Box(p \vee q)$ and $\Box p \vee \Box q$
   (d) $\Diamond(p \wedge q)$ and $(\Diamond p) \vee (\Diamond q)$
   (e) $\Box(p \to q)$ and $(\Box p) \to (\Box q)$

4. Prove that the following formulas are valid:

   (a) $\Box p \to (\Box q \to \Box p)$
   (b) $\Box(\varphi \wedge \psi) \to (\Box\varphi \to \Box\psi)$
   (c) $\Box(\varphi \wedge \psi) \to (\Diamond\varphi \to \Diamond\psi)$
   (d) $(\Diamond p \wedge \Box q) \to \Diamond(p \wedge q)$

5. Let $\mathcal{M} = \langle W, R, L\rangle$ and $\mathcal{M}' = \langle W', R', L'\rangle$ be two Kripke structures. A relation $Z \subseteq W \times W'$ is said to be *bisimulation* between $\mathcal{M}$ and $\mathcal{M}'$ if the following conditions are satisfied:

(a) For all $w \in W$ and $w' \in W'$ such that $(w, w') \in Z$ and each $p \in \mathcal{AP}$, it holds $p \in L(w)$ if and only if $p \in L'(w')$;
(b) If $(w, w') \in Z$ and $(w, v) \in R$, then there exists $v' \in W'$ such that $(v, v') \in Z$ and $(w', v') \in R'$ (*forth condition*);
(c) If $(w, w') \in Z$ and $(w', v') \in R$, then there exists $v \in W$ such that $(v, v') \in Z$ and $(w, v) \in R$ (*back condition*);

Worlds w and w' are called bisimilar, $w \leftrightsquigarrow_{\mathcal{M},\mathcal{M}'} w'$, if there is a bisimulation Z between $\mathcal{M}$ and $\mathcal{M}'$ with $(w, w') \in Z$.

An example of bisimulation is in Figure 1.

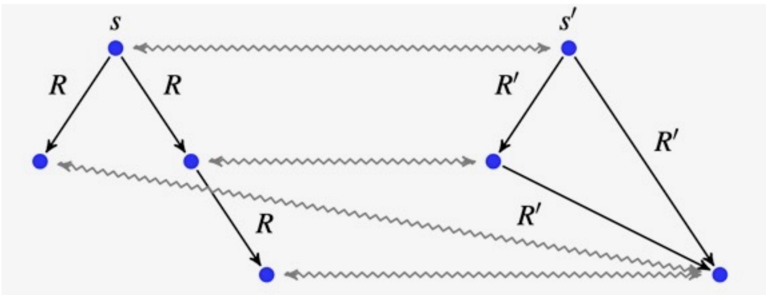

Figure 1: Example of bisimulation

Consider the two Kripke structures from Figure 2. Are the worlds s_0 and s'_0 bisimilar?

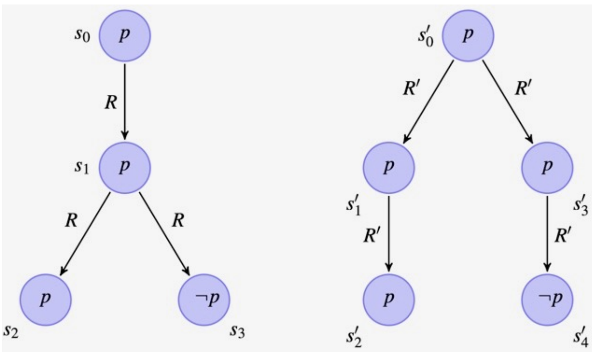

Figure 2: Example of bisimulation

6. Prove the following Proposition:

Proposition 1 *Let $\mathcal{M} = \langle W, R, L \rangle$ and $\mathcal{M}' = \langle W', R', L' \rangle$ be two Kripke structures. Let $w \in W$ and $w' \in W'$ such that $w \leftrightsquigarrow_{\mathcal{M},\mathcal{M}'} w'$. Then w and w' are modally equivalent, i.e., for each formula Modal Logic φ,*

$$\mathcal{M}, w \models \varphi \text{ iff } \mathcal{M}', w' \models \varphi$$