Homework part B.

15 points [3p: B1] + [4p: B2] + [3p: B3] + [5p: B4]

Deadline: week 1-6 June 2024 (the precise date is fixed by your lab teacher).

B1. (3 points) The volume of the ring torus

$$TC(R,r) = \left\{(x_1,x_2,x_3) \,:\, x_3^2 + \left(\sqrt{x_1^2+x_2^2} - R\right)^2 < r^2\right\} \subseteq [-R-r, R+r]\times[-R-r, R+r]\times[-r, r]$$

is $2\pi^2 Rr^2$. Estimate this volume using MC method for $R = 10, r = 3$, and compare the result with the exact value. Do this with samples of size 10000, 20000, and 50000, respectively; compute the corresponding relative errors.

B2. (4 points) Let T be the following triangle $T = \{(x,y) \,:\, y \geqslant 0, y \leqslant 2x, y \leqslant 6-3x\}$. Determine a rectangular area $[a,b]\times[c,d]$ that includes all of its interior points and then estimate the area covered by T using MC method with samples of size at least 20000.

B3. (3 points = 1p + 1p + 1p) Estimate the values of the following integrals and compare the results with the their exact values:

$$(a) \int_{-1}^{1} \frac{2x-1}{x^2-x-6}dx = \ln 3 - \ln 2;$$

$$(b) \int_{3}^{11} \frac{x+4}{\sqrt[3]{x-3}}dx = \lim_{a\to 3+}\left(\int_{a}^{11} \frac{x+4}{\sqrt[3]{x-3}}dx\right) = 61.2;$$

$$(c) \int_{0}^{\infty} xe^{-x^2}dx = \frac{1}{2}.$$

B4. (5 points = 2p + 1p + 2p) The new released social network iSocialize has in 2024 10000 users. Each year the network gains a number of new users that follows a binomial distribution $B(n,p)$; each of the existing users (at the begining of the year) independently leaves the network with probability q.

(a) Estimate the average number of years until iSocialize will have at least 15000 users.

(b) Estimate the probability that after 40 years and 10 months the network will have at least 15000 users.

(c) Estimate this probability, attaining the margin of error ± 0.01 with probability 0.99.

Use the following values: $n = 1000, p = 0.25, q = 0.01$.

Solutions to these exercises (the corresponding R functions and their calls) will be written in an single R script.

Probabilities&Statistics