Probabilities and Statistics

Probabilities and Statistics - Lecture 3

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Conditional version of the total probability formula

Probabilities and Statistics

Proposition 1

$$P(A|B) = P(C|B) \cdot P(A|B \cap C) + P(\overline{C}|B)P(A|B \cap \overline{C}),$$

if all conditioning events are possible.

proof:

$$P(C|B) \cdot P(A|B \cap C) + P(\overline{C}|B)P(A|B \cap \overline{C}) =$$

$$= \frac{P(B \cap C)}{P(B)} \cdot \frac{P(A \cap B \cap C)}{P(B \cap C)} + \frac{P(B \cap \overline{C})}{P(B)} \cdot \frac{P(A \cap B \cap \overline{C})}{P(B \cap \overline{C})} =$$

$$= \frac{P(A \cap B \cap C)}{P(B)} + \frac{P(A \cap B \cap \overline{C})}{P(B)} = \frac{P(A \cap B)}{P(B)} = P(A|B).$$

■

Conditional version of the total probability formula - example

Probabilities and Statistics

Example. A box contains two dice: one die (D_1) has number 4 on two of its faces, and the other, (D_2), is a fair one. We withdraw a die from the box and we toss it. If we get a 4, we toss again the same die, otherwise we chose the other die and we toss it.

a) What is the probability that the second toss gives a 4?

b) If the second toss gives a 4 what is the probability that the chosen die was D_1?

Solution. We denote by $A =$"the second toss gives a 4", $B =$ "the first chosen die is D_1", and by $C =$"the first toss gives a 4". a) For $P(A)$ we use the total probability formula

$$P(A) = P(B) \cdot P(A|B) + P(\overline{B}) \cdot P(A|\overline{B}).$$

Obviously, $P(B) = 1/2$.

Conditional version of the total probability formula - example

Probabilities and Statistics

For $P(A|B)$ and $P(A|\overline{B})$ we will use the conditional version of the total probability formula

$$P(A|B) = P(C|B) \cdot P(A|B \cap C) + P(\overline{C}|B)P(A|B \cap \overline{C}),$$

$$P(A|\overline{B}) = P(C|\overline{B}) \cdot P(A|\overline{B} \cap C) + P(\overline{C}|\overline{B})P(A|\overline{B} \cap \overline{C}),$$

$P(C|B) = 1/3, P(\overline{C}|B) = 2/3, P(A|B \cap C) = 1/3, P(A|B \cap \overline{C}) = 1/6,$

$P(C|\overline{B}) = 1/6, P(\overline{C}|\overline{B}) = 5/6, P(A|\overline{B} \cap C) = 1/6, P(A|\overline{B} \cap \overline{C}) = 1/3.$

Thus, $P(A|B) = 8/36, P(A|\overline{B}) = 11/36, P(A) = 19/72.$

b) For the second question we use the Bayes formula:

$$P(B|A) = \frac{P(B) \cdot P(A|B)}{P(A)} = \frac{8}{19}.$$

Multiplication rule

Probabilities and Statistics

Proposition 2

Let $A_1, A_2, \ldots, A_n$ *random events, then*

$$P(A_1 \cap A_2 \cap \ldots \cap A_n) =$$

$$= P(A_1) \cdot P(A_2 | A_1) \cdot \ldots \cdot P(A_n | A_1 \cap A_2 \cap \ldots \cap A_{n-1}),$$

when $P(A_1 \cap A_2 \cap \ldots \cap A_{n-1}) > 0$.

proof: The right member is equal with

$$P(A_1) \cdot \frac{P(A_1 \cap A_2)}{P(A_1)} \cdot \frac{P(A_1 \cap A_2 \cap A_3)}{P(A_1 \cap A_2)} \cdot \ldots$$

$$\ldots \cdot \frac{P(A_1 \cap A_2 \cap \ldots \cap A_{n-1} \cap A_n)}{P(A_1 \cap A_2 \cap \ldots \cap A_{n-1})} = P(A_1 \cap A_2 \cap \ldots \cap A_n)$$

(after some algebra). ■

Multiplication rule

Probabilities and Statistics

Example. A box contains 5 white balls and 5 black balls. We withdraw 3 balls one by one without replacement.

(a) What is the probability that all the balls are white?

(b) What is the probability that two balls are white and one is black?

Solution:

(a) For the first question let A_i ="the i th ball is white" ($i = \overline{1,3}$), then

$$P(A_1) = \frac{1}{2},\ P(A_2|A_1) = \frac{4}{9},\ P(A_3|A_1 \cap A_2) = \frac{3}{8},$$

therefore

$$P(A_1 \cap A_2 \cap A_3) = P(A_1) \cdot P(A_2|A_1) \cdot P(A_3|A_1 \cap A_2) = \frac{1}{2} \cdot \frac{4}{9} \cdot \frac{3}{8} = \frac{1}{12}.$$

Multiplication rule

Probabilities and Statistics

(b) For the second question the probability is

$$P\left(\left(\overline{A}_1 \cap A_2 \cap A_3\right) \cup \left(A_1 \cap \overline{A}_2 \cap A_3\right) \cup \left(A_1 \cap A_2 \cap \overline{A}_3\right)\right) =$$

$$P\left(\overline{A}_1 \cap A_2 \cap A_3\right) + P\left(A_1 \cap \overline{A}_2 \cap A_3\right) + P\left(A_1 \cap A_2 \cap \overline{A}_3\right).$$

Each of these probabilities can be computed using the multiplication rule. ♣

Multiplication rule

Probabilities and Statistics

Example. A box contains 4 white balls and 6 black balls. we withdraw 3 balls one by one without replacement.

(a) What is the probability that all the balls are black?

(b) What is the probability that the first and the third balls are white while the second is black?

(c) What is the probability that two balls are black and one is white?

Solution:

(a) For the first question let $A_i =$"the i-th ball is black".

$$P(A_1) = \frac{6}{10},\ P(A_2|A_1) = \frac{5}{9},\ P(A_3|A_1 \cap A_2) = \frac{4}{8} \text{ and}$$

$$P(A_1 \cap A_2 \cap A_3) = P(A_1) \cdot P(A_2|A_1) \cdot P(A_3|A_1 \cap A_2) = \frac{6}{10} \cdot \frac{5}{9} \cdot \frac{4}{8} = \frac{1}{6}.$$

Multiplication rule

Probabilities and Statistics

(b) For the second question the probability is

$$P\left(\overline{A}_1 \cap A_2 \cap \overline{A}_3\right) = P(\overline{A}_1) \cdot P(A_2|\overline{A}_1) \cdot P(\overline{A}_3|\overline{A}_1 \cap A_2),$$

and

$$P(\overline{A}_1) = \frac{4}{10}, P(A_2|\overline{A}_1) = \frac{6}{9}, P(\overline{A}_3|\overline{A}_1 \cap A_2) = \frac{3}{8}.$$

$$P\left(\overline{A}_1 \cap A_2 \cap \overline{A}_3\right) = \frac{4}{10} \cdot \frac{6}{9} \cdot \frac{3}{8} = \frac{1}{10}.$$

Multiplication rule

(c) For the third question the probability is

$$P\left(\left(\overline{A}_1 \cap A_2 \cap A_3\right) \cup \left(A_1 \cap \overline{A}_2 \cap A_3\right) \cup \left(A_1 \cap A_2 \cap \overline{A}_3\right)\right) =$$

$$P\left(\overline{A}_1 \cap A_2 \cap A_3\right) + P\left(A_1 \cap \overline{A}_2 \cap A_3\right) + P\left(A_1 \cap A_2 \cap \overline{A}_3\right)$$

all these probabilities will be computed using the multiplication rule. As an example

$$P(A_1 \cap \overline{A}_2 \cap A_3) = P(A_1) \cdot P(\overline{A}_2 | A_1) \cdot P(A_3 | A_1 \cap \overline{A}_2) = \frac{6}{10} \cdot \frac{4}{9} \cdot \frac{5}{8} = \frac{1}{6}. \clubsuit$$

Probabilities and Statistics

Hypergeometric Schema

Probabilities and Statistics

- This schema applies for the following framework: in a box we have n_1 white balls and n_2 black balls, $n = n_1 + n_2$. We withdraw from this box, without replacement, k balls.

Proposition 3

Let $k_1, k_2 \in \mathbb{N}$, *such that* $k_1 \leqslant n_1$, $k_2 \leqslant n_2$ *and* $k = k_1 + k_2$. *The probability that from those* k *balls, exactly* k_1 *are white and* k_2 *are black is*

$$\frac{\binom{n_1}{k_1} \cdot \binom{n_2}{k_2}}{\binom{n}{k}}.$$

Hypergeometric Schema

Probabilities and Statistics

- More general: in a box we have n_1 balls of color c_1, n_2 balls of color c_2, ..., n_p balls of color c_p (where $n = n_1 + n_2 + \cdots + n_p$). We withdraw from this box, without replacement, k balls.

Proposition 4

Let $k_1, k_2, \ldots, k_p \in \mathbb{N}$, such that $k_i \leqslant n_i$, $1 \leqslant i \leqslant p$ and $k = k_1 + k_2 + \cdots + k_p$. The probability that from those k balls, exactly k_1 are of color c_1, k_2 are of color c_2, ..., and k_p are of color c_p is

$$\frac{\binom{n_1}{k_1} \cdot \binom{n_2}{k_2} \cdot \ldots \cdot \binom{n_p}{k_p}}{\binom{n}{k}}.$$

Hypergeometric Schema

proof: Obviously the total number of combinations is $\binom{n_1+n_2}{k_1+k_2} = \binom{n}{k}$. Among these there are exactly $\binom{n_1}{k_1}$ possibilities to get exactly k_1 white balls, and for each of these possibilities there are another $\binom{n-n_1}{k-k_1} = \binom{n_2}{k_2}$ possibilities for getting $k - k_1 = k_2$ black balls.

■

Probabilities and Statistics

Hypergeometric Schema

Probabilities and Statistics

Example. In a box we have 4 white balls, 3 red balls, 5 black balls, and 4 blue balls. We withdraw from this box, without replacement, 7 balls.

(a) What is the probability that 2 of the balls are white, 3 are black, and 2 blue?

(b) What is the probability that exactly 4 of the balls are black?

Solution:

$$(a)\ \frac{\binom{4}{2}\cdot\binom{5}{3}\cdot\binom{3}{0}\cdot\binom{4}{2}}{\binom{16}{7}},$$

$$(b)\ \frac{\binom{5}{4}\cdot\binom{11}{3}}{\binom{16}{7}}.\clubsuit$$

Poisson Schema

Probabilities and Statistics

- The framework of this schema is the following: we consider a random experience and n independent random events (related to this experience): $A_1, A_2, \ldots, A_n$ with known probabilities:

$$P(A_1) = p_1, P(A_2) = p_2, \ldots, P(A_n) = p_n.$$

Proposition 5

Let $k \in \mathbb{N}$, $k \leqslant n$. The probability that exactly k events occur (from those n) equals the coefficient of x^k from the polynomial

$$(p_1 x + q_1) \cdot (p_2 x + q_2) \cdot \ldots \cdot (p_n x + q_n),$$

where $q_i = P(\overline{A_i}) = 1 - p_i$, $i = \overline{1, n}$.

Poisson Schema

Probabilities and Statistics

proof: Let $A_{i_1}, A_{i_2}, \ldots, A_{i_k}$ be the k events that occur ($1 \leqslant i_j \leqslant n$). In the same time will occur the events $\overline{A}_i$, with $i \in \{1, 2, \ldots, n\} \setminus \{i_1, i_2, \ldots, i_k\}$. These are independent events; the probability of their intersection is

$$\left(\prod_{j=1}^{k} p_{i_j}\right) \cdot \left(\prod_{i \notin \{i_1, i_2, \ldots i_k\}} q_i\right); \text{ we add all these probabilities:}$$

$$\sum_{1 \leqslant i_1 < i_2 < \ldots < i_k \leqslant n} \left(\prod_{j=1}^{k} p_{i_j}\right) \cdot \left(\prod_{i \notin \{i_1, i_2, \ldots i_k\}} q_i\right),$$

but this sum may be obtained in other way: by adding up all the products of the following form: for k factors of our polynomial we choose the coefficient of x, and for the rest we choose the free term (from the corresponding binomial). The result is the coefficient of x^k.

$\blacksquare$

Binomial Schema

Probabilities and Statistics

- We consider a random experience and a random event A with probability $P(A) = p$. We independently perform the experience n times.

Proposition 6

The probability that event A occurs exactly k times $(0 \leqslant k \leqslant n)$ is $p^k(1-p)^{n-k}\binom{n}{k}$.

Binomial Schema

Probabilities and Statistics

proof: Let the random experience be $\mathcal{E}$. We can define a new experiment $\mathcal{E}'$ like this: we independently perform $\mathcal{E}$, n times. We associate to this experiment the following random events: A_i - an event that occurs only if at the i-th performance of $\mathcal{E}$ the event A occurs.

We are, in this way, in the framework of Poisson schema, with n independent events $A_1, A_2, \ldots, A_n$ each having the same probability p. The required probability is the coeffcient of x^k in the development of the binomial $[px + (1-p)]^n$. ■

Binomial Schema

Probabilities and Statistics

Example. We roll two dice 10 times. What is the probability that the product of the two values is 12 exacly 6 times?
Solution: Let $A =$"the product is 12" (at one roll),

$$A = \{(2,6),(3,4),(4,3),(6,2)\},$$

therefore $P(A) = 4/36 = 1/9$. The probability that A occurs exactly 6 times is

$$\frac{1}{9^6} \cdot \frac{8^4}{9^4} \cdot \binom{10}{6} . ♣$$

Geometric Schema

Probabilities and Statistics

- Framework: we independently perform a random experiment until a designated event, A, occurs ($P(A) = p$).

Proposition 7

The probability that event A occurs first time at the n-th performance of the experience ($n \geqslant 1$) is $p(1-p)^{n-1}$.

proof: Let the random experience be $\mathcal{E}$. We can define a new random experiment $\mathcal{E}'$ like follows: we independently perform $\mathcal{E}$, n times. We associate to this experiment the following random events: A_i - an event that occurs only if at the i-th performance (from $\mathcal{E}'$) of $\mathcal{E}$ the event A occurs.

The probability that event A occurs first at the n-th performance of the experience is

$$P(\overline{A}_1 \cap \overline{A}_2 \cap \ldots \cap \overline{A}_{n-1} \cap A_n) = p(1-p)^{n-1}.$$

Geometric Schema

Probabilities and Statistics

Example. An urn contains 3 red balls, 2 black and 4 blue. We withdraw a ball and then we put it in the same urn until we get a blue or a red ball. What is the probability that only at the fourth withdrawal we get a red or a blue ball?

Solution: Let A ="the withdrawn ball is red or blue", $P(A) = 7/9$. The probability that event A occurs only at the fourth withdrawal:

$$\frac{7}{9} \cdot \frac{2^3}{9^3}. \clubsuit$$

Exercises for seminar

Probabilities and Statistics

- Conditional version of the total probability formula: I.1, I.2, I.4.
- Multiplication Formula: II.2, II.3, II.4.
- Probabilistic Schemata: III.3, III.4, IV.4, IV.8, V.1, V.7.
- Reserve: I.3, II.1, III.2, IV.7, V.4.

Probabilities and Statistics

End

Exercises - Conditional version of the total probability formula

Probabilities and Statistics

I.1. We have two boxes: B_1 containing three regular dice and one die having number 2 on exactly five faces and B_2 which contains two regular dice and one die having number 2 on exactly five faces. From B_1 we withdraw a die and we add it to U_2. Than we withdraw a die from B_2 and we toss it once. Let $A =$"after tossing the die we get number 2", $B =$"the withdrawn die from B_1 is a regular one" şi $C =$"the withdrawn die from B_2 is a regular one".

(a) Compute $P(A|B)$ and $P(A|\overline{B})$.

(b) Determine $P(A)$ and $P(B|A)$.

I.2. In a box there are two dice: one has number 3 on four of its faces and number 4 on the remaining two faces; the second die is a regular one. We choose at random a die from the box and we toss it. If we get number 3, then we toss the second die, otherwise we will toss again the first chosen die.

Exercises - Conditional version of the total probability formula

Probabilities and Statistics

(a) What is the probability that we get a 3 at the second toss?

(b) If at the second toss we get a number different from 3, what is the probability that the chosen die was the regular one?

I.3. A box contains two dice: one has number 3 on three of its faces and number 6 on the other three, while the other is a regular die. We choose at random a die and we roll it; if we get a 6 we roll it again, otherwise we roll the other die.

(a) What is the probability that we get a 6 at the second rolling?

(b) If at the second rolling we get a 6, what is the probability that the chosen die was the unfair one?

Exercises - Conditional version of the total probability formula

Probabilities and Statistics

I.4. We have three boxes: the first contains 3 white and 5 black balls, the second contains 3 white and 2 black balls, and the third contains 2 white and 4 black balls. We withdraw a ball from the first box and we add it to the second, than we withdraw a ball from the second box and we add it to the third, in the end we withdraw a ball from the last box. $A_i =$"the withdrawn ball from the ith box is white", $i = \overline{1,3}$.

(a) Compute $P(A_3)$.

(b) Determine $P(A_1|A_3)$.

Exercises - Total Probability and Bayes Formulas

Probabilities and Statistics

I.5*. k urns contains each p red balls and q blue balls. A ball is randomly chosen from the first urn and introduced in the second, then a ball is randomly chosen from the second urn and introduced in the third and so on. The final withdrawn is from the kth urn. Prove the the probability that the last ball is blue is the same as the probability that the first ball is blue.

I.6*. We have two boxes, one contains p white balls and the other p black balls ($p \geqslant 3$). An interchange consists in two simultaneous withdrawals of two balls, one from box 1 and one from box 2, after which the withdrawn ball is introduced in the other box.

(a) What is the probability that after two interchanges the boxes have the same content as before?

(b) What is the probability that after four interchanges the boxes have the same content as before?

Exercises - Multiplication Formula

Probabilities and Statistics

II.1. In a box we have 6 white balls, 4 blue, and 2 red. We withdraw three balls without replacement. What is the probability that

(a) the first ball is white, and the other two are blue?

(b) a ball is blue, and two are red?

II.2. In a box we have 3 white balls, 2 black, and 4 red. We withdraw three balls without replacement. What is the probability that

(a) all balls are red?

(b) the first ball is black and the last two are red?

Exercises - Multiplication Formula

Probabilities and Statistics

II.3. We have three urns. One contains 3 white balls and 1 black, the second contains 4 white balls and 5 black, and the last contains 1 white balls and 4 black. From the first urn a ball is withdrawn and transferred to the second, then from the second urn a ball is withdrawn and transferred to the third; at last a ball is withdrawn from the third urn. What is the probability that

- (a) all three balls are white?
- (b) two balls are black and one is white?
- (c) at least a ball is white?

II.4. A box contains five white balls and seven black balls. Anytime we withdraw a ball from the box we replace it with two new balls of the opposite colour. We make three such withdrawals. Find the probability that

- (a) the first two withdrawn balls are of different colours.
- (b) the first ball is white and the following two are black.

Exercises - Multiplication Formula

Probabilities and Statistics

II.5. Two balls are coloured with blue or green and then introduced in a box. A ball is green coloured with probability $1/3$.

(a) If we know that blue colour was used (that is, at least a ball is blue coloured), what is the probability that both balls are blue?

(b) The box is overthrown and a blue ball falls. What is the probability that the remaining ball is green?

Exercises - Hypergeometric Schema

Probabilities and Statistics

III.1. From a deck of (52) cards we withdraw four cards.

(a) What is the probability that exactly two of them are red?

(b) What is the probability that exactly one of them is black?

III.2. In a bag there are 4 black balls and 5 white balls. We withdraw 4 balls from the bag. What is the probability that

(a) two balls are white and two are black?

(b) all balls are black?

(c) a ball is white and three are black?

Exercises - Hypergeometric Schema

Probabilities and Statistics

III.3. In a box there are 3 red balls, 4 blue, and 5 green; four balls are withdrawn from the box. What is the probability that

(a) two balls are red, one is blue, and one is green?

(b) a ball is red, one is blue, and two are green?

(c) three balls are blue and one is green?

III.4. From a deck of (52) cards we withdraw seven cards.

(a) What is the probability that exactly two of them are odd numbers and three are figures?

(b) What is the probability that exactly three of them are numbers numbers of diamonds and two of them are figures of clubs? (Ace is not a figure nor a number.)

Exercises - Binomial Schema

IV.1. We flip one hundred coins.

(a) What is the probability that we get fifty tails?

(b) But the probability that we get at least fifty tails?

IV.2. HDD Ltd. company produces hard-disks that have malfunctions with probability of 0.05. The company sells its hd's in bunches of ten and guarantees that such a package contains at most one defective hd, otherwise the entire package will be replaced. What is the probability for a given package to be replaced?

IV.3. An archer athlete hit the target with a probability of 0.5; he shoots ten times. What is the probability that

a) the target is hit exactly five times?

a) the target is hit at least five times?

Exercises - Binomial Schema

Probabilities and Statistics

IV.4. We toss two dice ten times. What is the probability that

a) exactly five times the sum is greater or equal to 6, but the second die is different from 4?

b) at most eight times the sum is a prime number?

IV.5. Facing an equal opponent, what is more probable to win: two games out of four or three out of six?

IV.6. A communication channel randomly and independently transmits bits (0 with probability 0.25). The bits are received in pairs. If seven pairs of bits are transmitted, what is the probability that

a) exactly five times the received pair is an $0-1$?

b) at most six times we receive an $1-1$?

Exercises - Binomial Schema

Probabilities and Statistics

IV.7. A pair of dice is rolled six times. One die has number 3 on all faces, the other one being a fair one. What is the probability that

a) exactly four times the product of the dice is an even number?

b) at least four times the sum is a prime number?

IV.8. A pair of coins is flipped five times. One of the coins has the probability of getting head equal with $1/3$, the other one being a fair one. What is the probability that

a) exactly three times the coins have different faces?

b) at least four times we get head on both coins?

Exercises - Binomial Schema

Probabilities and Statistics

IV.9. A pair of dice is tossed six times. What is the probability that

a) exactly three times the minimum of the two values we get is 3?

b) at least four times the sum of the values is greater than 6?

IV.10*. In the framework of the binomial schema: an experience is independently performed n times, and we have in mind a random event A, $P(A) = p$, related with this experience; what number of occurrences of A is more probable?

Exercises - Geometric Schema

V.1. From a deck of 52 cards we randomly withdraw a card (which is then returned to the deck) until we get an ace. What is the probability that only at the fifth withdrawal we get an ace?

V.2. We toss two dice until we get a sum at least 8. What is the probability that

(a) this happens only at the third toss?

(b) this happens in one of the first two tosses?

V.3*. Two players successively roll two dice. The winner is the player get first a total at most 9. What are the chances of winning for the player which starts the game?

Exercises - Geometric Schema

Probabilities and Statistics

V.4. We toss two coins many times. What is the probability that

(a) the first time we get two heads is at the fourth toss?

(b) the first time we get exactly one head is at the fourth toss?

V.5. A card from a regular deck is withdrawn and then is returned to the deck. We perform this experience many times. What is the probability that

(a) the first time we get a face clubs is at the third withdrawal?

(b) in the first four withdrawals we get no diamonds?

V.6. A communication channel randomly and independently transmits bits (0 with probability 0.4). The bits are received in pairs. What is the probability that

(a) the first time we get an $1-0$ occurs with the fourth pair?

(b) the first time we get an $1-1$ occurs with the third pair?

Exercises - Geometric Schema

V.7. A pair of dice is rolled many times. One of the die has number 2 on all faces, the other one being a fair one. What is the probability that

a) the first time the product is a prime number occurs at the fourth roll?

b) the first time we get a double occurs at the third roll?

V.8. We toss a pair of coins many times. One of the coins has the probability of getting head equal with 1/4, the other being a fair one. What is the probability that

(a) the first time we get head on both coins occurs at the fifth toss?

(b) the first time we get tail on both coins occurs at the third toss?

Exercises - Geometric Schema

Probabilities and Statistics

V.9. A pair of dice is tossed many times. What is the probability that

(a) the first time the maximum of the two values we get is 5 occurs at the fourth toss?

(b) the first time the sum of the values is less than 6 occurs at the fifth toss?

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