

Homework part B.

5 points [1p: B1] + [1p: B2] + [3p: B3]

B1. (1 point) The volume of a triaxial ellipsoid

$$E(a, b, c) = \left\{ (x, y, z) : \frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} \leq 1 \right\} \subseteq [-a, a] \times [-b, b] \times [-c, c]$$

is $\frac{4}{3}\pi abc$. Estimate this volume by using Monte Carlo method for $a = 2, b = 3, c \in \{5, 6\}$ and compare the result with the true value. Use samples of size 10000, 20000, and 50000 and compute the relative errors.

B2. (1 point) Estimate the following integrals and compare the results with the true values:

$$(a) \int_0^{+\infty} \frac{\sin x}{x} dx = \lim_{a \rightarrow +\infty} \int_0^a \frac{\sin x}{x} dx = \frac{\pi}{2}; (b) \int_0^{+\infty} x e^{-x^2} dx = \frac{1}{2}.$$

B3. (3 points) Each day a number of AI-generated accounts occur in the social network Y and these accounts have to be deleted. This number of accounts in the i th day, denoted by X_i , is $Poisson(\min(X_{i-1}/2, X_{i-2} - 1))$. We suppose that in the first two days the number of this kind of accounts was 40 și 36, respectively.

- (a) (1 point) What is the expected number of days needed for all these accounts to be deleted?
- (b) (1 point) Estimate the probability that 5 days are enough for deleting all these accounts.
- (c) (1 point) Estimate this last probability, attaining the margin of error ± 0.01 with probability 0.99.

(Use $N = 1000$ runs for the Monte Carlo estimator.)