Homework part A.

5 points [2p: A1] + [2p: A2] + [1p: A3]

A1. (2 points) Laplace distribution has its probability density function (PDF) defined by

$$f(x) = \frac{1}{2b} \mathrm{e}^{-\frac{|x-\mu|}{b}},$$

where $\mu \in \mathbb{R}$ is a *location* parameter, and $b > 0$ is a *scale* parameter.

(a) (1 point) Simulate $n = 10000$ observations from Laplace distribution with the following parameters: $\mu = 0$, $b \in \{1/2, 1, 2, 4\}$.

(b) (1 point) For each set of parameters, plot the theoretical PDF and overlay a histogram of the above sample.

A2. (2 points) Verify (or explain the failure of) the law of large numbers (LLN) for each of the following cases:

(a) (1 point) Hypergeometric distribution with parameters $n_1 \in \{10, 40\}$, $n_2 \in \{7, 10\}$, $k = 5$ (see Lecture 5):

- Compute sample means for sample sizes 10^i, $i \in \{1, 2, 3, 4, 5\}$ and compare with $\mathbb{E}[X] = kn_1/(n_1 + n_2)$.
- Plot means vs. sample sizes.

(b) (1 point) Cauchy distribution with parameters $l = 0$, $s \in \{1, 2, 5\}$:

- Compute sample means for sample sizes 10^i, $i \in \{1, 2, 3, 4, 5\}$.
- Plot means vs. sample sizes.

A3. (1 point) Verify the central limit theorem (TLC) for the geometric distribution with $p = 0.25$, *i.e.*:

- For $n = 5, 10, 30, 50, 100$ (sample sizes), generate 10000 sample means;
- Standardize and compare the empirical cumulative distribution function (CDF) with the CDF of the normal distributions.

Hints: Samples of required random variables can be called in R by the following functions: rlaplace, rhyper, rcauchy, rgeom.

Solutions to these exercises (the corresponding R functions and their calls) will be written in a single R script.