Computer Vision

Course 6

Types of Noise and Their Characteristics

Noise = a variation of the signal from its true value by a small (random) amount due to external or internal factors in the image processing pipeline. (Solomon & Breckon, Fundamentals of Digital Image Processing)

Noise = unwanted information in the image

The noise may be correlated or uncorrelated, signal dependent or independent, and so on. The knowledge about the imaging system and the visual perception of the image helps in generating the noise model and estimating of the statistical characteristics of noise embedded in an image is important because it helps separating the noise from the useful image signal. We consider four classes of noise:

1. **Additive noise**: sometimes the noise generated from sensors are normal white Gaussian, which is essentially additive and signal independent,

$$g(x,y) = f(x,y) + \eta(x,y).$$

2. **Multiplicative noise**: the graininess noise from photographic plates is usually multiplicative.

$$g(x,y) = f(x,y) * \eta(x,y).$$

3. **Impulse noise**: often the noisy sensors genrate impulse noise:

$$g(x,y) = (1-p)f(x,y) + p\,i(x,y),$$

where ***i(x,y)*** is the impulsive noise and ***p*** is a binary parameter. The impulse noise can be easily detected from noisy images because of the contrast anomalies.

4. **Quantization noise**, is essentially an image dependent noise. This noise is characterized by the size of signal quantization interval. Such noise produces image-like artifacts and may produce false contours around the objects. The quantization noise also removes the image details which are of low contrast.

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Two dimensional images may be degraded due to several reasons, e.g.

- Imperfection of the imaging system
- Imperfection in the transmission channel
- Degradation due to atmospheric conditions
- Degradation due to relative motion between the object and the camera

Noise Models

$$g(x,y) = f(x,y) + \eta(x,y)$$

The main sources of noise in digital images arise during image acquisition and/or transmission (environmental conditions during image acquisition, the quality of the sensors).

Parameters that define the spatial characteristics of the noise and whether the noise is correlated with the image are important properties to be studied. We assume that the noise is independent of spatial coordinates and that it is uncorrelated with the image itself (i.e. there is no correlation between pixel values and the values of noise components).

Some Important Noise Probability Density Functions

The noise may be considered a random variable, characterized by a probability denisty function (pdf).

Gaussian noise

$$p(z) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(z-\bar{z})^2}{2\sigma^2}}$$

where z represents intensity, $\bar{z}$ is the mean value, and σ is its standard deviation, σ^2 is called variance of z.

Rayleigh noise

$$p(z) = \begin{cases} \frac{2}{b}(z-a)e^{-\frac{(z-a)^2}{b}} & \text{for } z \geq a \\ 0 & \text{for } z < a \end{cases}$$

The mean and variance for this pdf are:

$$\bar{z} = a + \sqrt{\frac{\pi b}{4}}$$

$$\sigma^2 = \frac{b(4-\pi)}{4}$$

Erlang (gamma) noise

$$p(z)=\begin{cases}\dfrac{a^b z^{b-1}}{(b-1)!}e^{-az} & \text{for } z\geq 0\\ 0 & \text{for } z<0\end{cases}\quad,\qquad a,b>0,\ b\in\mathbb{N}$$

$$\bar{z}=\frac{b}{a}$$

$$\sigma^2=\frac{b}{a^2}$$

Exponential noise

$$p(z)=\begin{cases} ae^{-az} & \text{for } z \geq 0 \\ 0 & \text{for } z < 0 \end{cases} \quad , \quad a > 0$$

$$\bar{z} = \frac{1}{a}$$

$$\sigma^2 = \frac{1}{a^2}$$

(Erlang ***b=1***)

Uniform noise

$$p(z)=\begin{cases}\dfrac{1}{b-a} & \text{for } a \le z \le b \\ 0 & \text{otherwise}\end{cases},$$

$$\bar{z}=\frac{a+b}{2}$$

$$\sigma^2=\frac{(b-a)^2}{12}$$

Impulse (salt-and-pepper) noise

The pdf of *(bipolar) impulse* noise is given by

$$p(z) = \begin{cases} P_a & \text{for } z = a \\ P_b & \text{for } z = b \\ 0 & \text{otherwise} \end{cases}$$

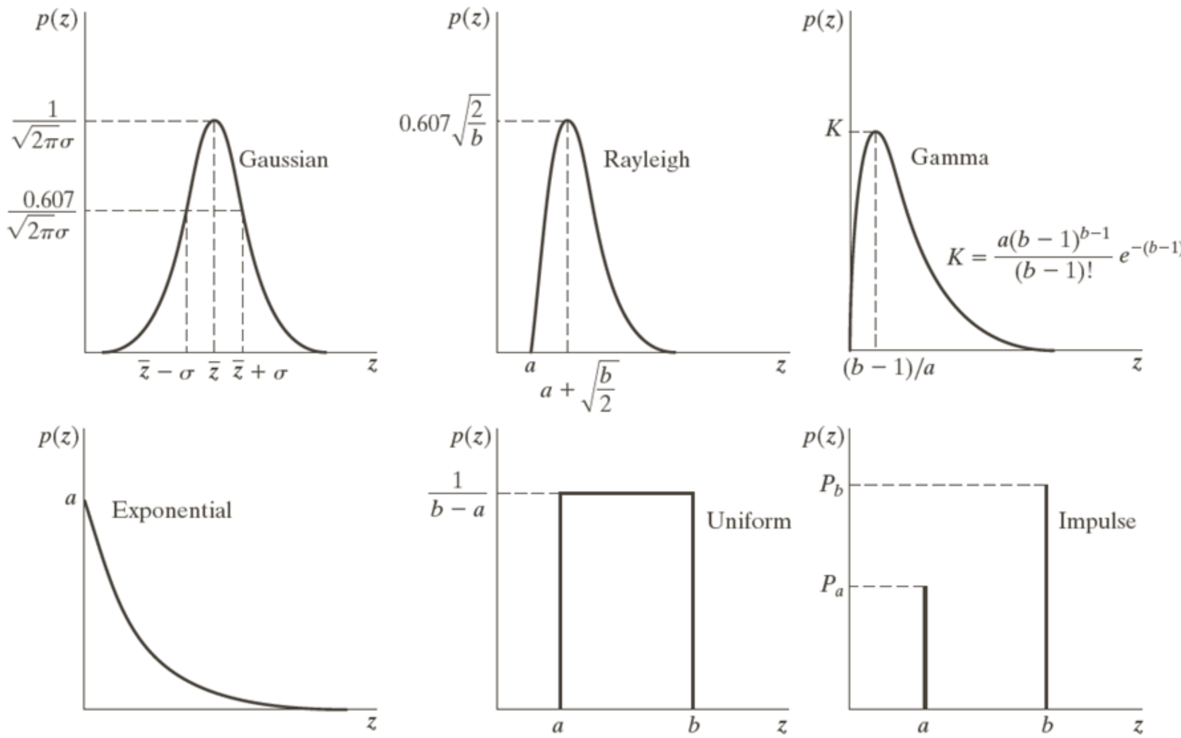
$b > a$ – intensity b appear as a light dot in the image

$b < a$ – intensity b appear as a dark dot in the image

$P_a = 0$ or $P_b = 0$ the impule noise is called *unipolar*

$\boldsymbol{P_a \approx P_b}$ - impulse noise values will ressemble salt and pepper granules randomly distributed over the image. For this reason, bipolar impulse noise is called also *salt-and-pepper* noise.

Noise impulses can be negative or positive. Because impulse corruption usually is large compared with the strength of the image signal, impulse noise generally is digitized as extreme (pure black or white) values in an image. Thus, the assumption is that ***a*** and ***b*** are equal to the minimum and maximum allowed values in the digitized image. As a result, negative impulses appear as black (pepper) points in an image, and positive impulses appear as white (salt) points.

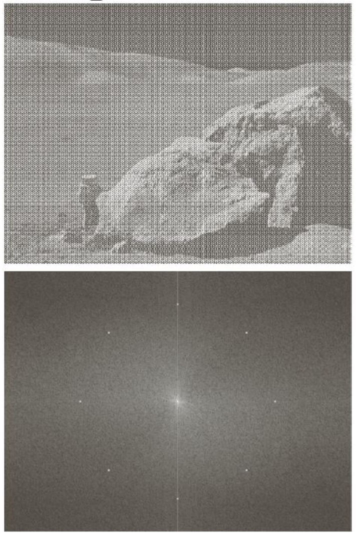


a b c
d e f

Some important probability density functions.

Periodic noise

Periodic noise arises from electrical or electromechanical interference during image acquisition. This type of noise is spatially dependent and can be reduced significantly via frequency domain filtering.



a
b
FIGURE 5.5 (a) Image corrupted by sinusoidal noise. (b) Spectrum (each pair of conjugate impulses corresponds to one sine wave). (Original image courtesy of NASA.)

Figure 5.5(a) is corrupted by sinusoidal noise of various frequencies. The Fourier transform of a pure sinusoid is a pair of conjugate impulses located at the conjugate frequencies of the sine wave. If the amplitude of a sine wave in the spatial domain is strong enough, we would expect to see in the spectrum of the image a pair of impulses for each sine wave in the image. In Figure 5.5(b) we can see the impulses appearing in a circle.

Gaussian noise – electronic circuit noise, sensor noise due to poor illumination and/or high temperature

Rayleigh noise – noise in radar range imaging

Exponential and gamma noise – laser imaging

Salt & pepper noise – „sharp and sudden disturbances in the image signal” (Wiki)

Uniform noise – theoretical use

Estimation of Noise Parameters

The parameters of periodic noise are estimated by inspection of the Fourier spectrum of the image. Sometimes it is possible to deduce the periodicity of noise just by looking at the image.

The parameters of noise pdf's may be known partially from sensors specifications. If the image system is available, one simple way to study the characteristics of system noise is to capture a set of images of „flat" environments (in the case of an optical sensor, this means taking images of a solid gray board that is illuminated uniformly). The resulting images are good indicators of system noise.

When only images already generated by a sensor ar available, frequently it is possible to estimate the parameters of the pdf from small portions of the image that are of constant background intensity.

The simplest use of the data from the image strips is for calculating the mean and the variance of intensity levels.

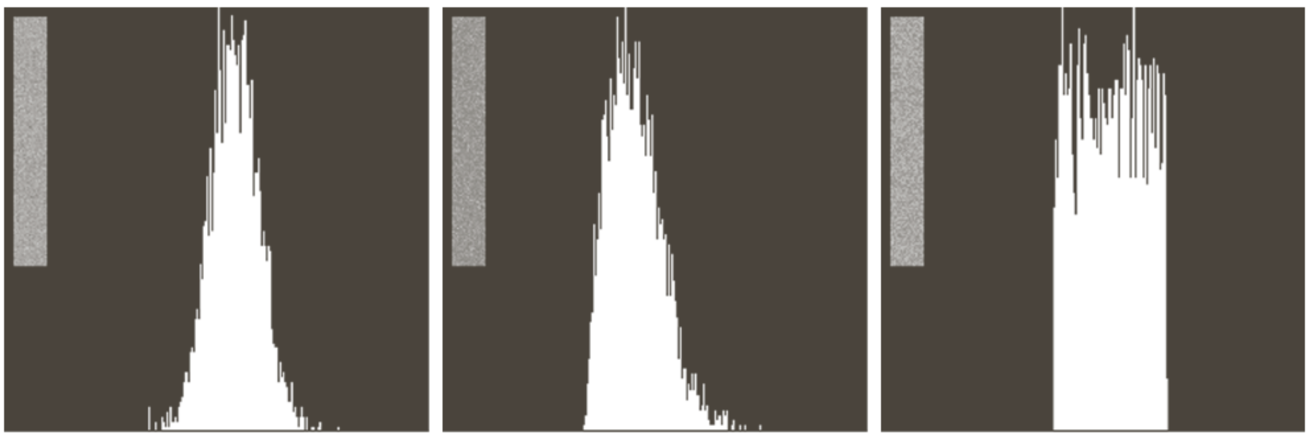
Consider a subimage ***S*** and let $p_S(z_i)$, $i=0,1,2,\ldots,L-1$ denote the probability estimates (normalized histogram values) of the intensities of the pixels in ***S***, where ***L*** is the number of possible intensities in the entire image. We estimate the mean and the variance of the pixels in ***S***:

$$\bar{z} = \sum_{i=0}^{L-1} z_i \, p_S(z_i)$$

$$\sigma^2 = \sum_{i=0}^{L-1} (z_i - \bar{z})^2 \, p_S(z_i)$$

The shape of the histogram identifies the closest pdf match. If the shape is almost Gaussian then the mean and the variance are all we need. For the other shapes, we use the mean and the variance to solve for parameters ***a*** and ***b***.

Impulse noise is handled differently because the estimate needed is of the actual probability of occurrence of white and black pixels. Obtaining this estimate requires that both black and white pixels be visible, so a midgray, relatively constant area is needed in the image in order to be able to compute a histogram. The heights of the peaks corresponding to black and white pixels are the estimates of P_a and P_b.



a b c

FIGURE 5.6 Histograms computed using small strips (shown as inserts) from (a) the Gaussian, (b) the Rayleigh, and (c) the uniform noisy images in Fig. 5.4.

Denoising filters

Mean Filters

Suppose $\boldsymbol{S_{xy}}$ represent a recatngular neighborhood of $\boldsymbol{m\times n}$ size centered at point ***(x,y)***.

Arithmetic mean filter

$$\hat{f}(x,y) = \frac{1}{m \cdot n} \sum_{(s,t)\in S_{xy}} g(s,t)$$

A mean filter smooths local variations of an image and noise is reduced as a result of blurring.

Geometric mean filter

$$\hat{f}(x,y) = \left[\prod_{(s,t)\in S_{xy}} g(s,t) \right]^{\frac{1}{m\cdot n}}$$

A geometric mean filter achieves smoothing comparable to the arithmetic mean filter, but it tends to lose less image detail.

Harmonic mean filter

$$\hat{f}(x,y) = \frac{m \cdot n}{\sum_{(s,t) \in S_{xy}} \frac{1}{g(s,t)}}$$

Harmonic mean filter works well for salt noise, but fails for pepper noise. It also works well on Gaussian noise.

Contraharmonic mean filter

$$\hat{f}(x,y)=\frac{\sum_{(s,t)\in S_{xy}} g(s,t)^{Q+1}}{\sum_{(s,t)\in S_{xy}} g(s,t)^{Q}} \quad , \quad Q - \text{the order of the filter}$$

This filter is good for reducing or virtually eliminating the effects of salt-and-pepper noise.

For $Q > 0$ the filter eliminates pepper noise, for $Q < 0$ the filter eliminates salt noise, but it cannot do both simultaneously.

$Q = 0$ – arithmetic mean filter, $Q = -1$ – harmonic mean filter

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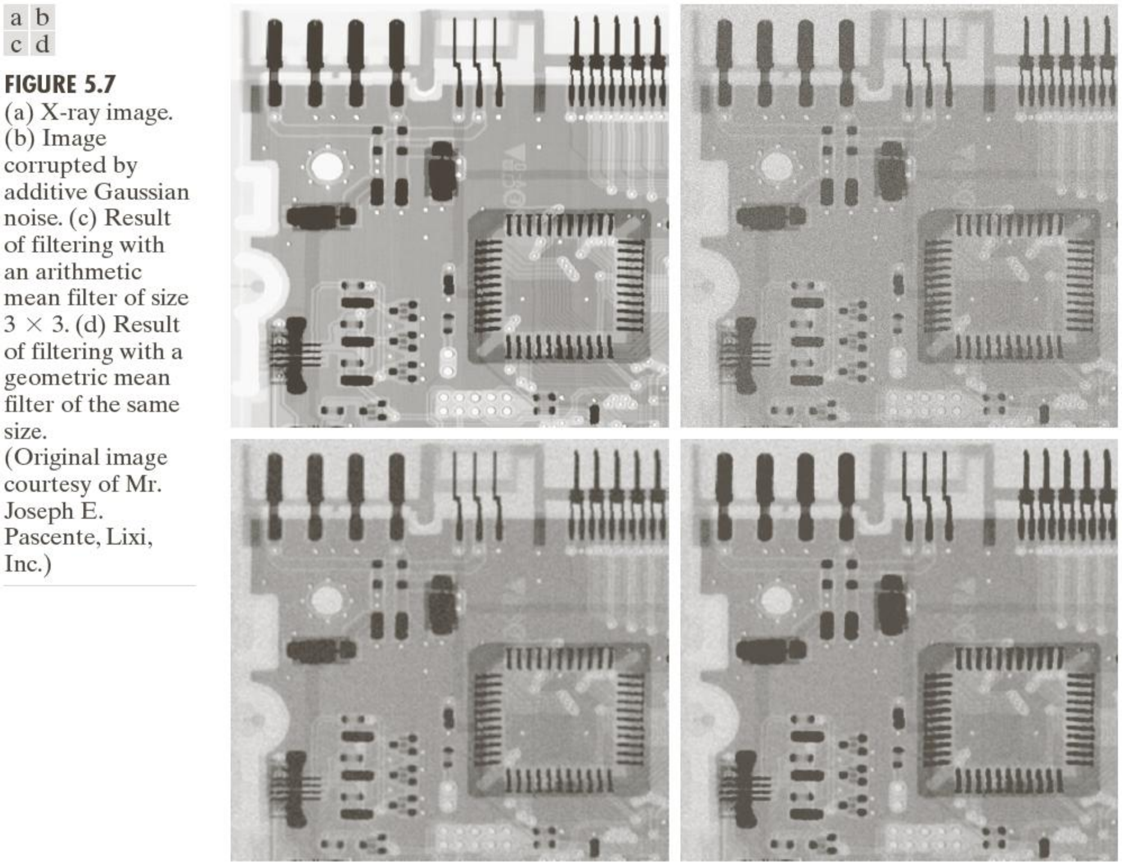

FIGURE 5.7 (a) X-ray image. (b) Image corrupted by additive Gaussian noise. (c) Result of filtering with an arithmetic mean filter of size 3×3. (d) Result of filtering with a geometric mean filter of the same size. (Original image courtesy of Mr. Joseph E. Pascente, Lixi, Inc.)

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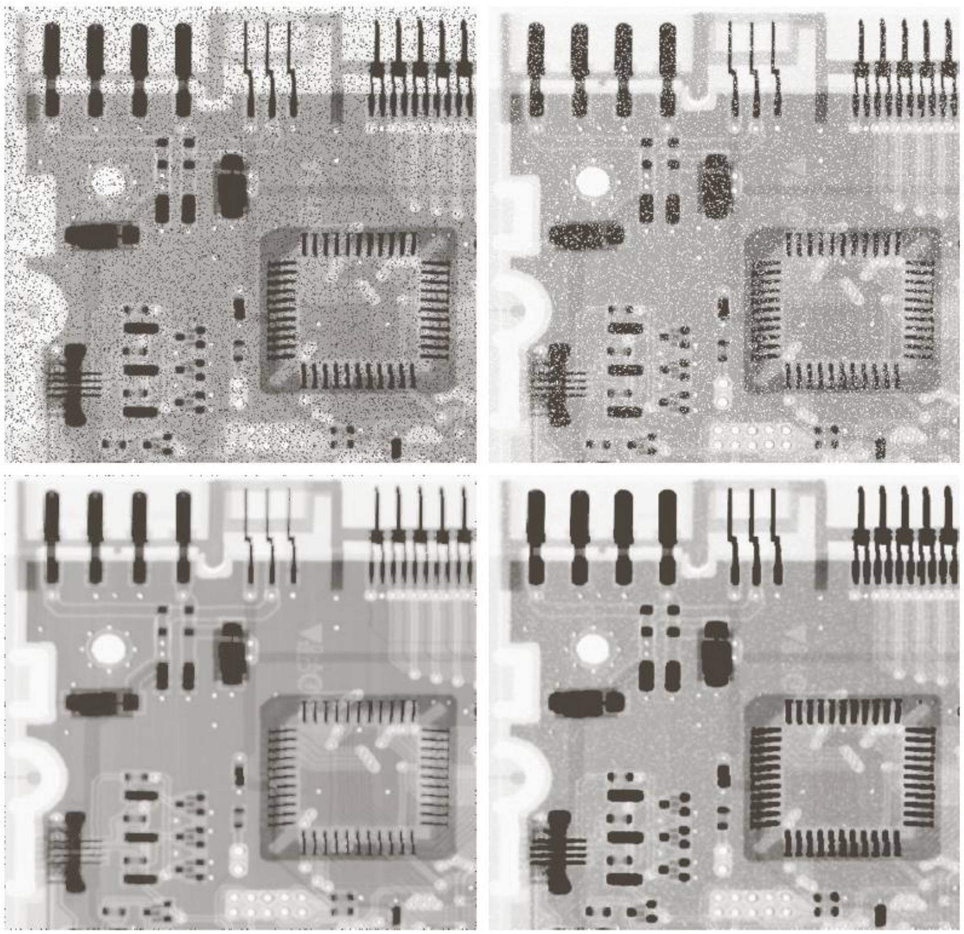

a b
c d

FIGURE 5.8 (a) Image corrupted by pepper noise with a probability of 0.1. (b) Image corrupted by salt noise with the same probability. (c) Result of filtering (a) with a 3×3 contra-harmonic filter of order 1.5. (d) Result of filtering (b) with $Q = -1.5$.

a b

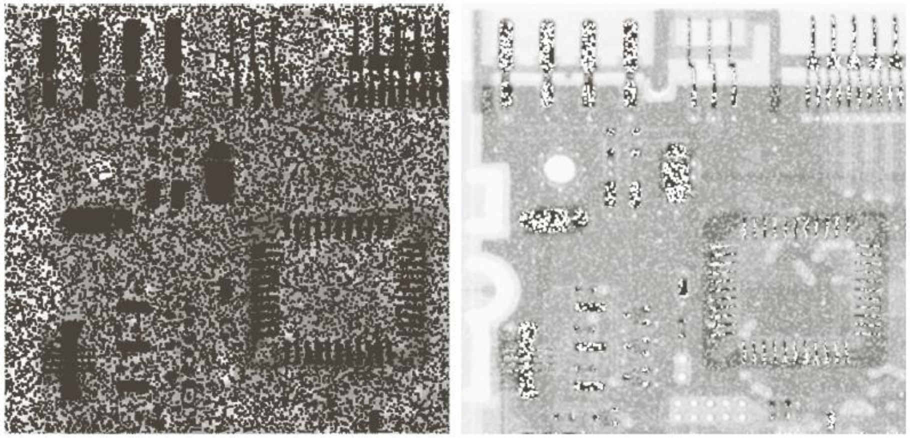

FIGURE 5.9 Results of selecting the wrong sign in contraharmonic filtering. (a) Result of filtering Fig. 5.8(a) with a contraharmonic filter of size 3 × 3 and $Q = -1.5$. (b) Result of filtering 5.8(b) with $Q = 1.5$.

Order-Statistic Filters

Median filter

$$\hat{f}(x,y) = \text{median}\{g(s,t);(s,t) \in S_{xy}\}$$

Median filters have excellent noise-reduction capabilities, with less blurring than linear smoothing filters. Median filters are particularly effective in the presence of bipolar and unipolar impulse noise.

Max and min filters

$$\hat{f}(x, y) = \max\{g(s,t);(s,t) \in S_{xy}\}$$

This filter is useful for finding the brightest points in an image. This filter reduces pepper noise.

$$\hat{f}(x, y) = \min\{g(s,t);(s,t) \in S_{xy}\}$$

This filter is useful for finding the darkest points in an image. This filter reduces salt noise.

Midpoint filter

$$\hat{f}(x,y)=\frac{1}{2}\left[\max\{g(s,t);(s,t)\in S_{xy}\}+\min\{g(s,t);(s,t)\in S_{xy}\}\right]$$

It works best for randomly distributed noise, like Gaussian or uniform noise.

Adaptive, Local Noise Reduction Filter

g(x,y) – noisy image, S_{xy} – neighborhood, $\boldsymbol{\sigma_{\eta}^{2}}$ - the variance of the noise, m_{xy} – local avergage intensities in S_{xy}, $\boldsymbol{\sigma_{xy}^{2}}$ - local variance in S_{xy}

Properties of the desired filter:

1. If $\boldsymbol{\sigma_{\eta}^{2} = 0}$, the filter return *g(x, y)* (no noise).
2. If the local variance $\boldsymbol{\sigma_{xy}^{2}}$ is high relative to $\boldsymbol{\sigma_{\eta}^{2}}$ the filter should return a value close to *g(x, y)*. A high local variance

typically is associated with edges, and these should be preserved.

3. If $\sigma_{xy}^2 \approx \sigma_\eta^2$ the filter returns m_{xy}. This condition occurs when the local area has the same properties as the overall image, and local noise is to be reduced by averaging.

$$\hat{f}(x,y) = g(x,y) - \frac{\sigma_\eta^2}{\sigma_{xy}^2}\left(g(x,y) - m_{xy}\right)$$

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Morphological Image Processing

Morphology deals with form and structure. *Mathematical morphology* is a tool for extracting image components that are useful in the representation and description of region shape, such as boundaries, skeletons, and the convex hull. In this chapter, the inputs are binary images but the outputs are attributes extracted from these images.

$$A = \{(i, j);\ I_{i,j} = 1(\text{white})\} \subset \mathbb{Z}^2$$

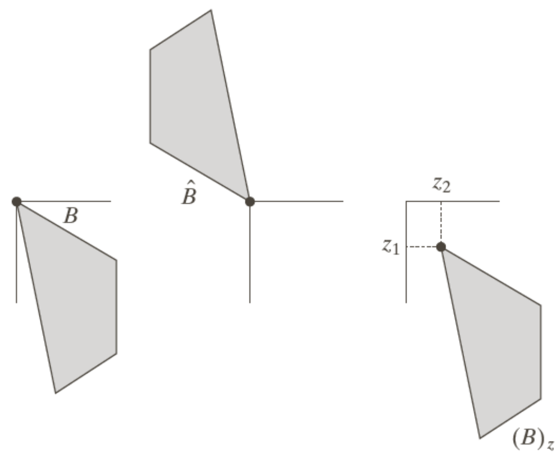
Preliminaries

The *reflection* of a set $\boldsymbol{B}$, denoted $\bar{\boldsymbol{B}}$ is defined as

$$\bar{B} = \{ w \; ; \; w = -b \; , \text{ for } \; b \in B \; \}$$

The *translation* of a set $\boldsymbol{B}$ by point $z = (z_1, z_2)$, denoted $(B)_z$ is defined as

$$(B)_z = \{ c \, ; \, c = z + b \text{ for } b \in B \; \}$$



a b c

FIGURE 9.1 (a) A set, (b) its reflection, and (c) its translation by z.

Set reflection and translation are used in morphology to formulate operations based on so-called *structuring elements* (SE): small sets or subimages used to probe an image under study for properties of interest. In addition to a definition of which elements are members of the SE, the origin of a structuring element also must be specified. The origin of the SE is usually indicated by a black dot. When the SE is symmetric and no dot is shown, the assumption is that the origin is at the center of symmetry.

When working with images, it is required that structuring elements to be rectangular arrays. This is accomplished by appending the smallest possible number of background elements necessary to form a rectangular array.

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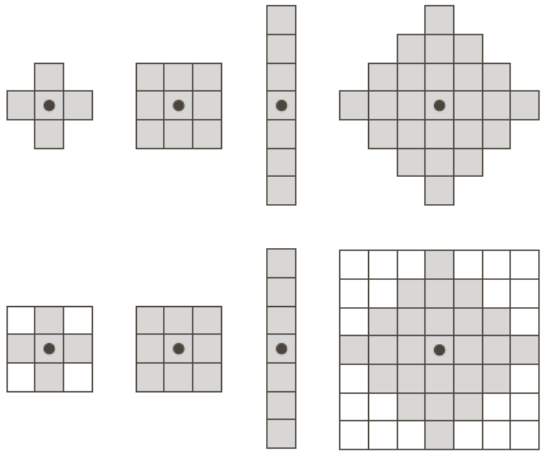

FIGURE 9.2 First row: Examples of structuring elements. Second row: Structuring elements converted to rectangular arrays. The dots denote the centers of the SEs.

Erosion and Dilation

Many of the morphological algorithms are based on these two primitive operations: *erosion* and *dilation*.

Erosion

Let $\boldsymbol{A}$ and $\boldsymbol{B}$ be to sets from $\mathbb{Z}^2$. The *erosion* of $\boldsymbol{A}$ by $\boldsymbol{B}$, denoted $\boldsymbol{A} \odot \boldsymbol{B}$ is defined as:

$$\boldsymbol{A} \odot \boldsymbol{B} = \{ \boldsymbol{z} \,;\, (\boldsymbol{B})_{\boldsymbol{z}} \subseteq \boldsymbol{A} \}$$

This definition indicates that the erosion of $\boldsymbol{A}$ by $\boldsymbol{B}$ is the set of all points $\boldsymbol{z}$ such that $\boldsymbol{B}$, translated by $\boldsymbol{z}$, is contained in $\boldsymbol{A}$. In the following, set $\boldsymbol{B}$ is assumed to be a structuring element. Because the statement that ***B*** has to be contained in ***A*** is equivalent to ***B*** not shearing any common elements with the background, erosion can be expressed equivalently:

$$\boldsymbol{A} \odot \boldsymbol{B} = \{ \boldsymbol{z} \,;\, (\boldsymbol{B})_{\boldsymbol{z}} \cap \boldsymbol{A}^{c} = \varnothing \}$$

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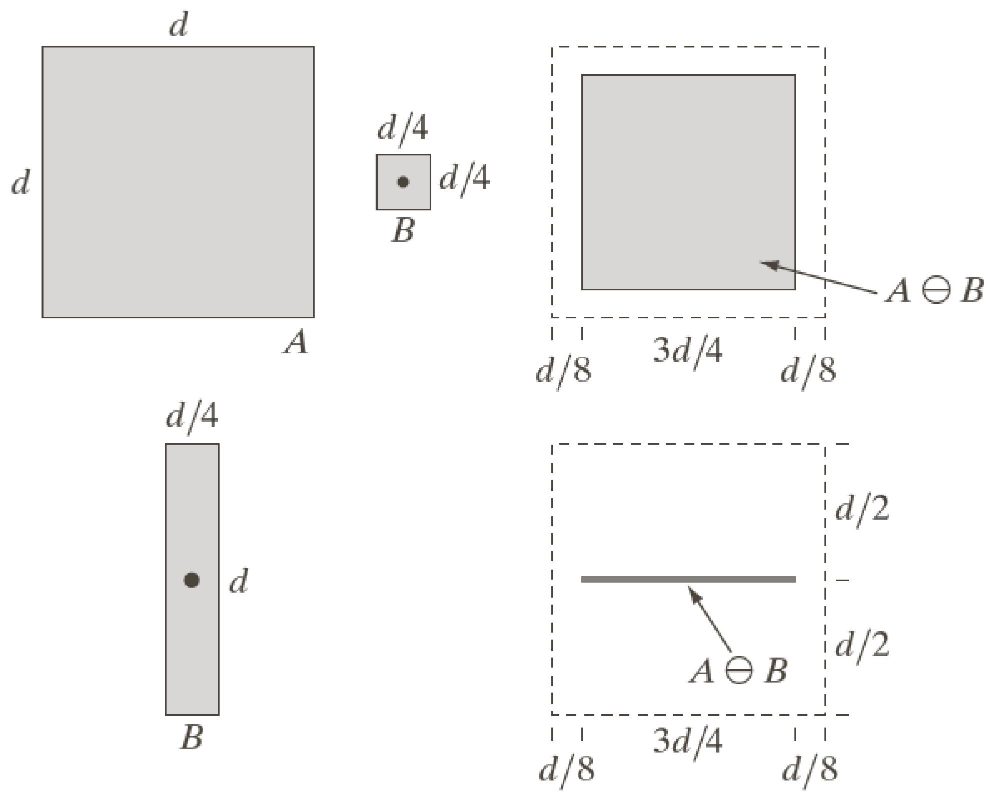


FIGURE 9.4 (a) Set A. (b) Square structuring element, B. (c) Erosion of A by B, shown shaded. (d) Elongated structuring element. (e) Erosion of A by B using this element. The dotted border in (c) and (e) is the boundary of set A, shown only for reference.

Equivalent definitions of erosion:

$$\boldsymbol{A} \odot \boldsymbol{B} = \{ \boldsymbol{w} \in \mathbb{Z}^2 ; \boldsymbol{w} + \boldsymbol{b} \in \boldsymbol{A} \quad \text{for every} \ \boldsymbol{b} \in \boldsymbol{B} \}$$

$$\boldsymbol{A} \odot \boldsymbol{B} = \bigcap_{\boldsymbol{b} \in \boldsymbol{B}} (\boldsymbol{A})_{-\boldsymbol{b}}$$

Erosion shrinks or thins objects in a binary image. We can view erosion as a *morphological filtering* operation in which image details smaller than the structuring element are filtered (removed) from the image.

Dilation

Let $\boldsymbol{A}$ and $\boldsymbol{B}$ be to sets in $\mathbb{Z}^2$. The *dilation* of $\boldsymbol{A}$ by $\boldsymbol{B}$, denoted $\boldsymbol{A} \oplus \boldsymbol{B}$ is defined as:

$$\boldsymbol{A} \oplus \boldsymbol{B} = \{\ \boldsymbol{z}\ ;\ (\bar{\boldsymbol{B}})_z \cap \boldsymbol{A} \neq \varnothing\ \}$$

The dilation of $\boldsymbol{A}$ by $\boldsymbol{B}$ is the set of all displacements $\boldsymbol{z}$, such that $\bar{\boldsymbol{B}}$ and $\boldsymbol{A}$ overlap by at least one element.

We assume that $\boldsymbol{B}$ is a structuring element.

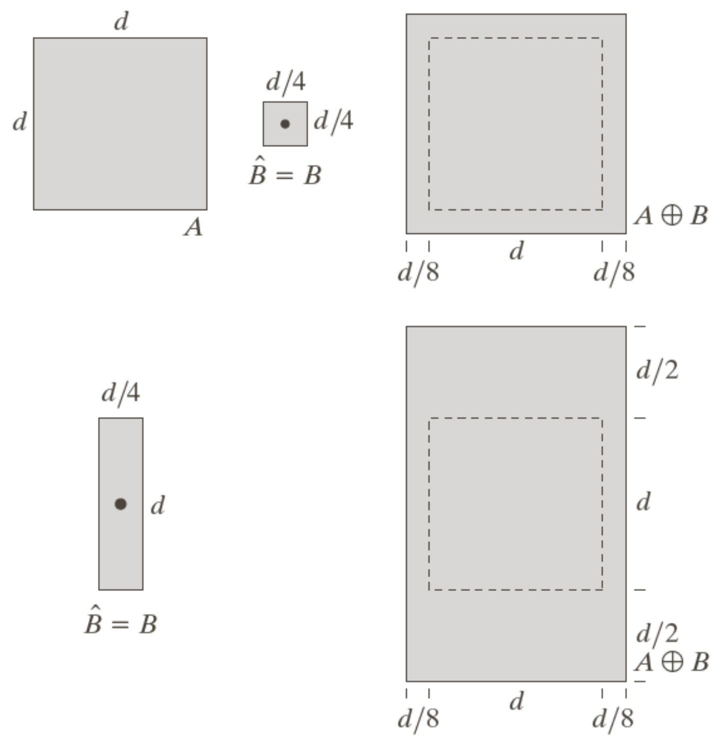
Equivalent definitions of dilation:

$$\boldsymbol{A} \oplus \boldsymbol{B} = \{\ \boldsymbol{w} \in \mathbb{Z}^2\ ;\ \boldsymbol{w} = \boldsymbol{a} + \boldsymbol{b}\ ,\ \text{for some}\ \ \boldsymbol{a} \in \boldsymbol{A}\ \text{and}\ \boldsymbol{b} \in \boldsymbol{B}\ \}$$

$$A \oplus B = \bigcup_{b \in B} (A)_b$$

The basic process of rotating ***B*** about its origin and then successively displacing it so that it slides over set (image) ***A*** is analogous to spatial convolution. Dilation being based on set operations is a nonlinear operation, whereas convolution is a linear operation.

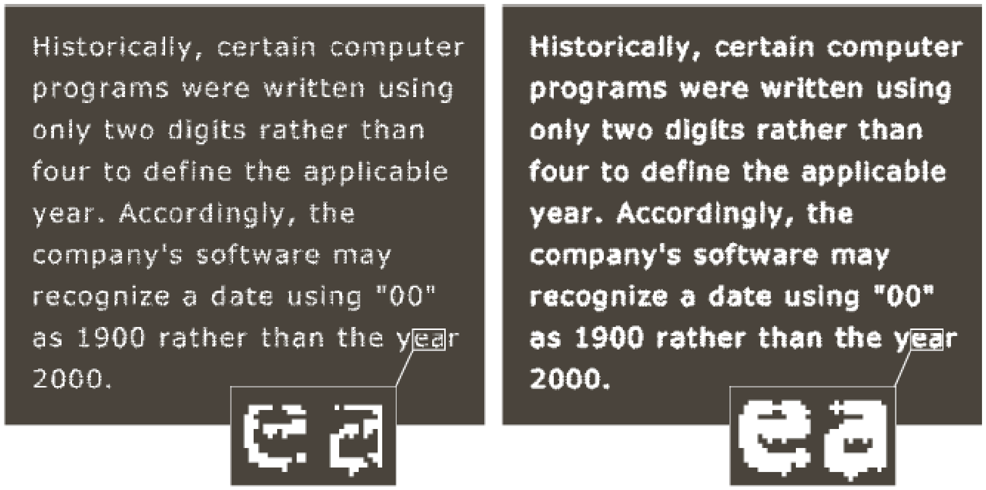
Unlike erosion which is a shrinking or thinning operation, dilation "grows" or "thickens" objects in a binary image. The specific manner and the extent of this thickening are controlled by the shape of the structuring element used.



a b c
d e

FIGURE 9.6 (a) Set A. (b) Square structuring element (the dot denotes the origin). (c) Dilation of A by B, shown shaded. (d) Elongated structuring element. (e) Dilation of A using this element. The dotted border in (c) and (e) is the boundary of set A, shown only for reference

One of the simplest applications of dilation is for bridging gaps.



0	1	0
1	1	1
0	1	0

a c
b

FIGURE 9.7 (a) Sample text of poor resolution with broken characters (see magnified view). (b) Structuring element. (c) Dilation of (a) by (b). Broken segments were joined.

Duality

Erosion and dilation are duals of each other with respect to set complementation and reflection:

$$\left(A \odot B\right)^c = A^c \oplus \bar{B}$$

$$\left(A \oplus B\right)^c = A^c \odot \bar{B}$$

The duality property is useful particularly when the structuring element is symmetric with respect to its origin, so that $B = \bar{B}$. Then, we can obtain the erosion of an image by B simply by dilating its background (i.e. dilating A^c) with the same structuring element and complementing the result.

Opening and Closing

The *Opening* operation generally smooths the contour of an object, breaks narrow isthmuses, and eliminates thin protrusions.

Closing also tends to smooth section of contours but, as opposed to opening, it generally fuses narrow breaks and long thin gulfs, eliminates small holes, and fills gaps in the contour.

The *opening* of set ***A*** by structuring element ***B*** is defined as:

$$\boldsymbol{A} \circ \boldsymbol{B} = (\boldsymbol{A} \odot \boldsymbol{B}) \oplus \boldsymbol{B}$$

Thus, the opening of ***A*** by ***B*** is the erosion of ***A*** by ***B***, followed by a dilation of the result by ***B***.

Similarly, the *closing* of set $\boldsymbol{A}$ by structuring element $\boldsymbol{B}$ is defined as:

$$\boldsymbol{A} \bullet \boldsymbol{B} = \left(\boldsymbol{A} \oplus \boldsymbol{B} \right) \odot \boldsymbol{B}$$

which says that the closing of $\boldsymbol{A}$ by $\boldsymbol{B}$ is the dilation of $\boldsymbol{A}$ by $\boldsymbol{B}$, followed by an erosion of the result by $\boldsymbol{B}$.

The opening operation has a simple geometric interpretation. Suppose that we view the structuring element $\boldsymbol{B}$ as a (flat) “rolling ball”. The *boundary* of $\boldsymbol{A} \circ \boldsymbol{B}$ is then established by the points in $\boldsymbol{B}$ that reach the farthest into the boundary of A as $\boldsymbol{B}$ is rolled around the inside of this boundary. The opening of ***A*** by ***B*** is obtained by taking the union of all translates of ***B*** that fit into ***A***.

$$\boldsymbol{A \circ B = \bigcup \{ (B)_z, z \in A; (B)_z \subseteq A \}}$$

Closing has a similar geometric interpretation, except that now we roll ***B*** on the outside of the boundary.

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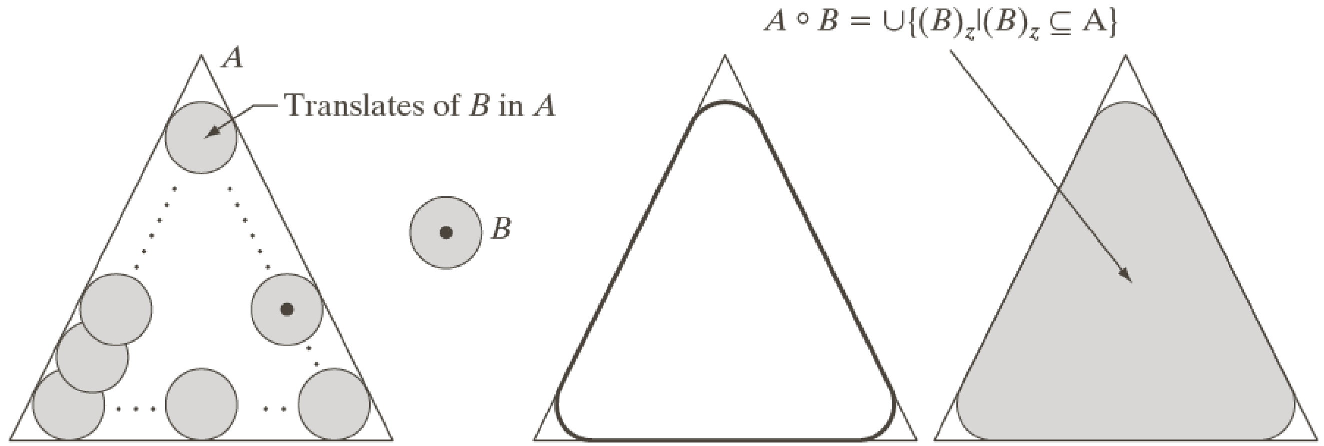

a b c d

FIGURE 9.8 (a) Structuring element B "rolling" along the inner boundary of A (the dot indicates the origin of B). (b) Structuring element. (c) The heavy line is the outer boundary of the opening. (d) Complete opening (shaded). We did not shade A in (a) for clarity.

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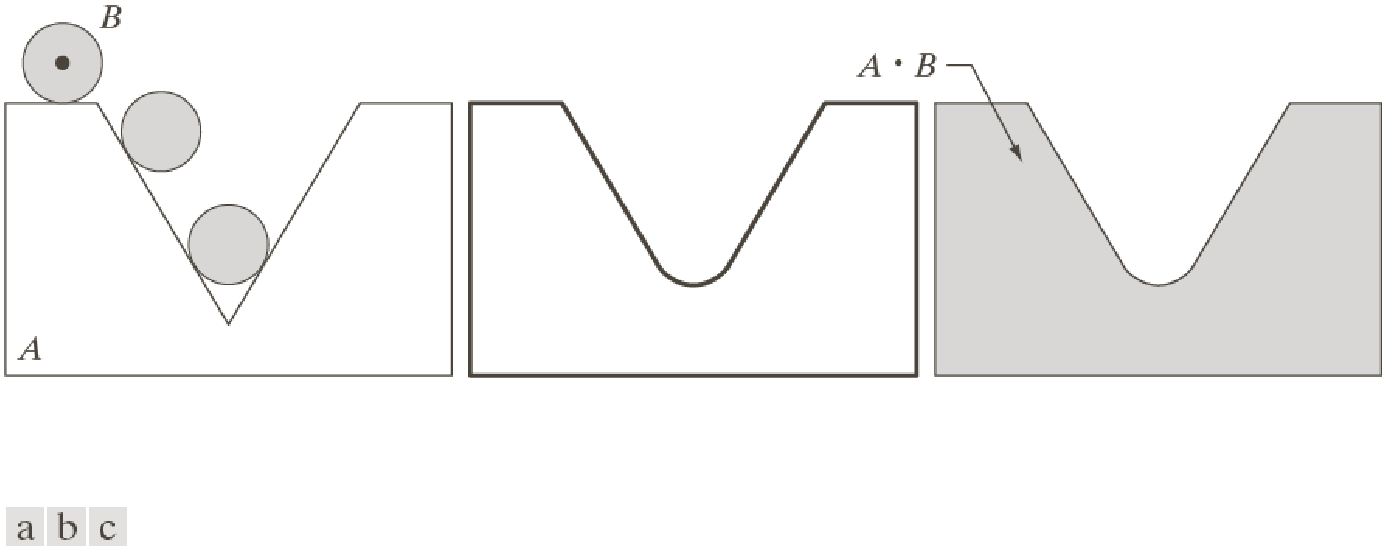

FIGURE 9.9 (a) Structuring element *B* "rolling" on the outer boundary of set *A*. (b) The heavy line is the outer boundary of the closing. (c) Complete closing (shaded). We did not shade *A* in (a) for clarity.

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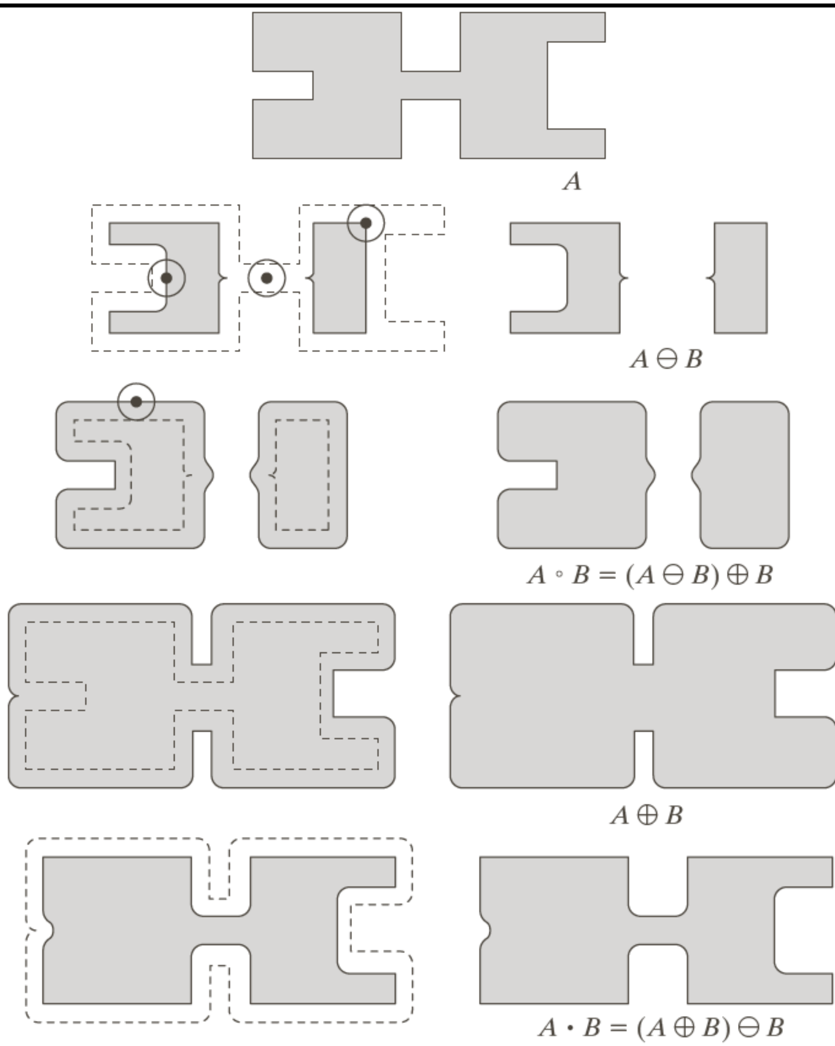

FIGURE 9.10 Morphological opening and closing. The structuring element is the small circle shown in various positions in (b). The SE was not shaded here for clarity. The dark dot is the center of the structuring element.

Opening and closing are dual of each other with respect to set complementation and reflection:

$$\left(A \bullet B\right)^c = A^c \circ \bar{B}$$

$$\left(A \circ B\right)^c = A^c \bullet \bar{B}$$

The opening operation satisfies the following properties:

1. $A \circ B \subseteq A$
2. if $C \subseteq D$ then $C \circ B \subseteq D \circ B$
3. $\left(A \circ B\right) \circ B = A \circ B$

Similarly, the closing operation satisfies the following properties:

1) $\boldsymbol{A} \subseteq \boldsymbol{A} \bullet \boldsymbol{B}$

2) if $\boldsymbol{C} \subseteq \boldsymbol{D}$ then $\boldsymbol{C} \bullet \boldsymbol{B} \subseteq \boldsymbol{D} \bullet \boldsymbol{B}$

3) $\left(\boldsymbol{A} \bullet \boldsymbol{B}\right) \bullet \boldsymbol{B} = \boldsymbol{A} \bullet \boldsymbol{B}$

Condition 3 in both cases states that multiple openings or closings of a set have no effect after the operator has been applied once.

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a b
d c
e f

FIGURE 9.11
(a) Noisy image.
(b) Structuring element.
(c) Eroded image.
(d) Opening of A.
(e) Dilation of the opening.
(f) Closing of the opening.
(Original image courtesy of the National Institute of Standards and Technology.)