Seminar 6

1. Dual Simplex algorithm

Consider the following LP problem

$$\begin{cases} \min & 5x_1 + 2x_2 + 8x_3 \\ \text{s. t.} & 2x_1 - 3x_2 + 2x_3 \geqslant 3 \\ & -x_1 + x_2 + x_3 \geqslant 5 \\ & x_1, x_2, x_3 \geqslant 0 \end{cases}$$

We convert it in the standard form by subtracting excess variables:

$$\begin{cases} \min & 5x_1 + 2x_2 + 8x_3 \\ \text{s. t.} & 2x_1 - 3x_2 + 2x_3 - x_4 = 3 \\ & -x_1 + x_2 + x_3 - x_5 = 5 \\ & x_1, x_2, x_3, x_4, x_5 \geqslant 0 \end{cases}$$

We multiply the constraints by (-1)

$$\begin{cases} \min & 5x_1 + 2x_2 + 8x_3 \\ \text{s. t.} & -2x_1 + 3x_2 - 2x_3 + x_4 = -3 \\ & x_1 - x_2 - x_3 + x_5 = -5 \\ & x_1, x_2, x_3, x_4, x_5 \geqslant 0 \end{cases}$$

Table 1: First Dual Simplex tableau

	x_1	x_2	x_3	x_4	x_5	RHS
x_4	-2	3	-2	1	0	-3
x_5	1	-1	-1	0	1	-5
z	5	2	8	0	0	0
	$\left\lvert\frac{5}{-2}\right\rvert$ min		$\left\lvert\frac{8}{-2}\right\rvert$			

Table 2: Second Dual Simplex tableau

	x_1	x_2	x_3	x_4	x_5	RHS
x_1	1	-1.5	1	-.5	0	1.5
x_5	0	.5	-2	.5	1	-6.5
z	0	9.5	3	2.5	0	-7.5
			$\left\lvert\frac{3}{-2}\right\rvert$ min			

Table 3: Third Dual Simplex tableau

	x_1	x_2	x_3	x_4	x_5	RHS
x_1	1	-1.25	0	-.25	.5	-1.75
x_3	0	-.25	1	-.25	-.5	3.25
z	0	10.25	0	3.25	1.5	-17.25
		$\left\lvert\frac{10.25}{-1.25}\right\rvert$ min		$\left\lvert\frac{3.25}{-.25}\right\rvert$		

Table 4: Fourth Dual Simplex tableau

	x_1	x_2	x_3	x_4	x_5	RHS
x_2	-.8	1	0	.2	.4	1.4
x_3	-.2	0	1	-.8	-.6	3.6
z	8.2	0	0	1.2	5.6	-31.6

This last tableau is feasible, hence dual and primal optimal. The optimal solution is $x_1 = 0, x_2 = 1.4, x_3 = 3.6, x_4 = x_5 = 0$, the optimal value of the objective function is $z = 31.6$ (don't forget to change the sign here).

2. The following tableau corresponds to an iteration of the Dual Simplex method (using Bland's rule) when solving a minimization LP problem:

	x_1	x_2	x_3	x_4	x_5	x_6	RHS
	0	0	1	α_1	-1	0	α_2
	α_3	1	0	α_4	α_5	0	α_6
	-2	0	α_7	-1	-3	1	4
z	4	α_8	0	0	3	0	10

Categorize the variables as basic and nonbasic, label the rows, and provide the current values of all variables. Find the constraints on the parameters α_i so that the following statements are true.

(i) The above tableau is a valid tableau for the dual simplex algorithm.

(ii) The current basis is primal feasible.

(iii) The current basis is not primal feasible and x_4 enters the basis.

(iv) The current basis is not primal feasible and x_5 enters the basis.

(v) x_1 enters the basis, and the resulting solution is primal feasible (hence dual optimal).

(vi) The problem is infeasible.

3. Consider the following LP primal/dual pair of problems

$$(P)\left\{\begin{array}{lrl} \min & z = -7x_1 - 6x_2 - 5x_3 + 2x_4 - 3x_5 & \\ \text{s. t.} & x_1 + 3x_2 + 5x_3 - 2x_4 + 2x_5 & \leqslant 4 \\ & 4x_1 + 2x_2 - 2x_3 + x_4 + x_5 & \leqslant 3 \\ & 2x_1 + 4x_2 + 4x_3 - 2x_4 + 5x_5 & \leqslant 5 \\ & 3x_1 + x_2 + 2x_3 - x_4 - 2x_5 & \leqslant 1 \\ & x_1, \dots, x_4 & \geqslant 0, \end{array}\right.$$

$$(D)\left\{\begin{array}{lrl} \max & w = 4y_1 + 3y_2 + 5y_3 + y_4 & \\ \text{s. t.} & y_1 + 4y_2 + 2y_3 + 3y_4 & \leqslant -7 \\ & 3y_1 + 2y_2 + 4y_3 + y_4 & \leqslant -6 \\ & 5y_1 - 2y_2 + 4y_3 + 2y_4 & \leqslant -5 \\ & -2y_1 + y_2 - 2y_3 - y_4 & \leqslant 2 \\ & 2y_1 + y_2 + 5y_3 - 2y_4 & \leqslant -3 \\ & y_1, \dots, y_5 & \leqslant 0, \end{array}\right.$$

The solution: $x_1 = 0, x_2 = 4/3, x_3 = 2/3, x_4 = 5/3, x_5 = 0$ has been proposed as an optimal one for (P). Verify this using complementary slackness.