Seminar 3

1. Simplex algorithm

Consider the following LP problem

$$\begin{cases} \max & 10x_1 - 57x_2 - 9x_3 - 24x_4 + 2 \\ & 0.5x_1 - 5.5x_2 - 2.5x_3 + 9x_4 \leqslant 0 \\ & 0.5x_1 - 1.5x_2 - 0.5x_3 + x_4 \leqslant 0 \\ & x_1 \leqslant 1 \\ & x_1, x_2, x_3, x_4 \geqslant 0 \end{cases}$$

We convert it in the standard form

$$\begin{cases} \min & z = -10x_1 + 57x_2 + 9x_3 + 24x_4 - 2 \\ & 0.5x_1 - 5.5x_2 - 2.5x_3 + 9x_4 + x_5 = 0 \\ & 0.5x_1 - 1.5x_2 - 0.5x_3 + x_4 + x_6 = 0 \\ & x_1 + x_7 = 1 \\ & x_1, x_2, x_3, x_4, x_5, x_6, x_7 \geqslant 0 \end{cases}$$

Table 1: First Simplex tableau.

	x_1	x_2	x_3	x_4	x_5	x_6	x_7	RHS		
x_5	[0.5]	-5.5	-2.5	9	1	0	0	0	0/0.5	← min
x_6	0.5	-1.5	-0.5	1	0	1	0	0	0/0.5	
x_7	1	0	0	0	0	0	1	1	1/1	
z	-10	57	9	24	0	0	0	2		

Table 2: Second Simplex tableau.

	x_1	x_2	x_3	x_4	x_5	x_6	x_7	RHS		
x_1	1	-11	-5	18	2	0	0	0		
x_6	0	[4]	2	-8	-1	1	0	0	0/4	← min
x_7	0	11	5	-18	-2	0	1	1	1/11	
z	0	-53	-41	204	20	0	0	2		

Table 3: Third Simplex tableau.

	x_1	x_2	x_3	x_4	x_5	x_6	x_7	RHS		
x_1	1	0	[0.5]	-4	-0.75	2.75	0	0	0/0.5	← min
x_2	0	1	0.5	-2	-0.25	0.25	0	0	0/0.5	
x_7	0	0	-0.5	4	0.75	-2.75	1	1		
z	0	0	-14.5	98	6.75	13.25	0	2		

Table 4: Fourth Simplex tableau.

	x_1	x_2	x_3	x_4	x_5	x_6	x_7	RHS		
x_3	2	0	1	-8	-1.5	5.5	0	0		
x_2	-1	1	0	[2]	0.5	-2.5	0	0	0/2	← min
x_7	1	0	0	0	0	0	1	1		
z	29	0	0	-18	-15	93	0	2		

Table 5: Fifth Simplex tableau.

	x_1	x_2	x_3	x_4	x_5	x_6	x_7	RHS		
x_3	-2	4	1	0	[0.5]	-4.5	0	0	0/0.5	← min
x_4	-0.5	0.5	0	1	0.25	-1.25	0	0	0/0.25	
x_7	1	0	0	0	0	0	1	1		
z	20	9	0	0	-10.5	70.5	0	2		

Table 6: Sixth Simplex tableau.

	x_1	x_2	x_3	x_4	x_5	x_6	x_7	RHS		
x_5	-4	8	2	0	1	-9	0	0		
x_4	.5	-1.5	-.5	1	0	[1]	0	0	0/1	← min
x_7	1	0	0	0	0	0	1	1		
z	-22	93	-21	0	0	-24	0	2		

Table 7: Seventh Simplex tableau.

	x_1	x_2	x_3	x_4	x_5	x_6	x_7	RHS
x_5	-5	-5.5	-2.5	9	1	0	0	0
x_6	.5	-1.5	-.5	1	0	1	0	0
x_7	1	0	0	0	0	0	1	1
z	-10	57	9	24	0	0	0	2

The seventh tableau has the same content as the first tableau: we walked through a cycle. If we use Bland's rule (among all candidates for entering/leaving choose that having the smallest index) we can avoid this issue.

Table 8: Sixth' Simplex tableau.

	x_1	x_2	x_3	x_4	x_5	x_6	x_7	RHS		
x_5	-4	8	2	0	1	-9	0	0		
x_4	[0.5]	-1.5	-0.5	1	0	1	0	0	0/0.5	← min
x_7	1	0	0	0	0	0	1	1	1/1	
z	-22	93	21	0	0	-24	0	2		

Table 9: Seventh' Simplex tableau.

	x_1	x_2	x_3	x_4	x_5	x_6	x_7	RHS		
x_5	0	-4	-2	8	1	-1	0	0		
x_1	1	-3	-1	2	0	2	0	0		
x_7	0	3	[1]	-2	0	-2	1	1	1/1	← min
z	0	27	-1	44	0	20	0	2		

Table 10: Eighth' Simplex tableau.

	x_1	x_2	x_3	x_4	x_5	x_6	x_7	RHS
x_5	0	2	0	4	1	-5	2	2
x_1	1	0	0	0	0	0	1	1
x_3	0	3	1	-2	0	-2	1	1
z	0	30	0	42	0	18	1	3

The current basic feasible solution is an optimal one: $x_1 = 1, x_2 = 0, x_3 = 1, x_4 = 0, x_5 = 2, x_6 = x_7 = 0$; the objective function optimal value is -3.

2. The following tableau corresponds to an iteration of the simplex method:

	x_1	x_2	x_3	x_4	x_5	x_6	x_7	RHS
	1	α_1	0	α_2	0	2	0	1
	0	-3	0	-1	0	1	α_3	1
	0	1	1	α_4	0	4	0	α_5
	0	0	0	-2	1	4	0	1
z	0	α_6	0	α_7	α_8	1	0	12

Categorize the variables as basic and nonbasic and provide the current values of all the variables. Find teh constraints on the parameters α_i so that the following statements are true.

(i) The current basis is not optimal.

(ii) The current basis is optimal.

(iii) The current basis is optimal and alternative optimal bases exists.

(iv) The current basis is the unique optimal basis.

(v) The problem is unbounded.

(vi) When x_2 enters the basis, the change in the objective is zero.

3. Solve the following LP problem using the simplex method, then solve it using the geometric approach and outline the progress of the simplex algorithm.

$$\begin{cases} \min & z = x_1 - x_2 \\ \text{s. t.} & x_1 - x_2 \leqslant 2 \\ & x_1 + x_2 \leqslant 6 \\ & x_1, x_2 \geqslant 0 \end{cases}$$