

Seminar 10

1. Solve the following games and find their value: try first by eliminating weak dominated strategies, then look for a saddle point (maximin/minimax), and, only after that, try with mixed strategies (if the optimal strategies are mixed, just formulate the corresponding linear problems).

(a)

		B:		
		1	2	3
A:	1	1	1	4
	2	1	1	-1
	3	1	0	2
	4	4	3	1

(b)

		B:		
		1	2	3
A:	1	-1	1	4
	2	1	2	0
	3	3	0	1
	4	3	2	1

(c)

		B:			
		1	2	3	4
A:	1	0	-2	1	2
	2	1	-1	1	3
	3	2	0	-1	1
	4	3	-1	2	1

2. Consider the following three-person cooperative games having the payoffs ($v(\emptyset) = 0$):

(a)

S	{1}	{2}	{3}	{1, 2}	{1, 3}	{2, 3}	{1, 2, 3}
$v(S)$	1	1	0.75	2	2.5	3	4

(b)

S	{1}	{2}	{3}	{1, 2}	{1, 3}	{2, 3}	{1, 2, 3}
$v(S)$	0.75	1	0.5	2.5	2.5	3	3.5

Describe the cores of the games as a set of linear inequalities and compute the Shapley values for these games.

3. Assume there are three students, A , B , and C they have to study five OR lectures, OR_1, \dots, OR_5 , for their OR exam. Student A has the lectures OR_1 and OR_4 , student B has lectures OR_1, OR_3 and OR_4 , and student C has lectures OR_2, OR_3 , and OR_5 . When they are together students gather all their lectures and read them all. They cooperate in order to get the highest grade possible. It is supposed that their grade is proportional to the number of lectures they read (or two times the number of read lectures). Find a model for this story as a cooperative game.

(a) Find the characteristic function.

(b) Is this game superadditive? But cohesive? But essential?

- (c) Describe the core of the game.
- (d) Compute the Shapley value.

4. A glove-market game has four players: player A has two left-hand gloves, player B has a left-hand glove, player C has two right-hand gloves, and player D each have a right-hand glove. The worth of a coalition is the amount that it will get for the gloves in its possession. Every pair of gloves (left and right) can be sold in the market for \$4. Find a model for this game as a cooperative one.

- (a) Find the characteristic function.
- (b) Is this game superadditive? But cohesive? But essential?
- (c) Describe the core of the game.
- (d) Compute the Shapley value.

5. Consider the following three-person cooperative game having the payoffs ($v(\emptyset) = 0$):

S	{1}	{2}	{3}	{1, 2}	{1, 3}	{2, 3}	{1, 2, 3}
$v(S)$	1	1	0.75	2	2.5	3	4

- (a) **(4p)** Is this game superadditive? But cohesive? But essential?
- (b) **(4p)** Describe the core of the game.
- (c) **(4p)** Compute the Shapley value.