Seminar 1

Modeling.

1. (A Production Problem) A company produces n different goods using m different raw materials. Let b_i the available amount of the ith raw material ($i = \overline{1,m}$). The jth good requires a_{ij} units of the ith material and results in a revenue c_j per unit produced ($j = \overline{1,n}$). The company has to decide how much of each good to produce in order to maximize its total revenue.

2. (Pattern Classification - Support Vector Machine) We would like to separate m white points from n black points by a straight line in the plan (if possible). More formally, we are given m white points $p_1, p_2, \ldots p_m$ and n black points $q_1, q_2, \ldots q_n$ in the plane, and we would like to find out whether there exists a line having all white points on one side and all black points on the other side (some of the points may lie on the line). If such a line doesn't exists what we can do? Can you generalize this problem for more than two dimensions?

3. (Largest Disk in a Convex Polygon) Find the largest disk (the radix and the center coordinates) which lies entirely in a given convex polygon.

(*Hint*: the distance from a point (x_1, y_1) to line $ax + by + c = 0$ is $\dfrac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$.)

4. (The Optimal Assignment Problem) There are n persons available for m jobs. The value of person i working one day at job j is a_{ij}, for $i = \overline{1,n}, j = \overline{1,m}$. The problem is to choose an assignment of persons to jobs to maximize the total value.

5. There are n production plants, $P_1, P_2, \ldots, P_n$, that supply a certain commodity, and there are m markets, $M_1, M_2, \ldots, M_m$, to which this commodity must be shipped. Plant P_i possesses an amount s_i of the commodity ($i = \overline{1,n}$), and market M_j must receive the amount r_j of the commodity ($j = \overline{1,m}$). Let b_{ij} be the cost of transporting one unit of the commodity from plant P_i to market M_j. The problem is to meet the market requirements at minimum transportation cost.

6. Consider a school district with n neighborhoods, m schools, and p grades at each school. Each school $\mathcal{S}_j$ has capacity of c_{jh} for grade $\mathcal{G}_h$, $j = \overline{1,m}, h = \overline{1,p}$. In each neighborhood $\mathcal{N}_i$, the student population of grade $\mathcal{G}_h$ is n_{ih}. The distance of school $\mathcal{S}_j$ from neighborhood $\mathcal{N}_i$ is d_{ij}. Find an assignment of all students to schools, while minimizing the total distance traveled by students. (We can ignore the fact that the numbers of students must be integers.)

Modelling and solving.

7. (Activity Analysis or Product Mix) A lumber mill saws both finish-grade and construction-grade boards from the logs that it receives. Suppose that it takes 2 hr to rough-saw each 1000 board feet of the finish- or construction-grade boards and 5 hr to plane each 1000 board feet of the finish-grade boards. Suppose also that it takes only 3 hr to plane each 1000 board feet of construction-grade boards. The saw is available 8 hr per day, and the plane is available 15 hr per day. If the profit on each 1000 board feet of finish-grade boards is \$120 and the profit on each 1000 board feet of construction-grade boards is \$100, how many board feet of each type of lumber should be sawed and planed to maximize the profit in one single day?

8. (Internet Service Provider Problem) An ISP can provide two different services x_1 and x_2. The first service is, say, the fixed monthly rate with limited download limits and bandwidth, while the second service is the higher rate with no download limit. The rate of the profit is α for the first service, and β for the second ($\beta > \alpha$). Suppose that the provided service is limited by the total bandwidth of the ISP company, thus at most n_1 (in 1000 units) of the first and at most n_2 (in 1000 units) of the second can be provided per unit of time, say, each day. The management of each of the two service packages take the same staff time, so that a maximum of n (in 1000 units) can be maintained. Find x_1 and x_2 such that the profit is maximized for $\alpha = 2, \beta = 3, n_1 = 16, n_2 = 10, n = 20$.

9. A large textile firm has two manufacturing plants, two sources of raw material, and three market centers. The transportation costs between the sources and the plants and between the plants and the markets are as follows:

	Plant A	Plant B
Source 1	1	1.5
Source 2	2	1.5

	Market m_1	Market m_2	Market m_3
Plant A	4	2	1
Plant B	3	4	2

10 tons of raw material are available from source 1 and 15 tons from source 2. The three market centers require 8 tons, 14 tons, and 3 tons. The plants have unlimited processing capacity. The problem is to find the amounts of raw material and product that must be transported in order to minimize the transport costs.

10. A manufacturer wishes to produce an alloy that is, by weight, 40% metal A and 60% metal B. Four alloys are available at various prices as indicated below:

Alloy	A_1	A_2	A_3	A_4
$\%A$	30	50	75	80
$\%B$	70	50	25	20
Price/lb	4$	3$	3$	5$

The desired alloy will be produced by combining some of the other alloys. The manufacturer wishes to find the amounts of the various alloys needed and to determine the least expensive combination.

Geometric (graphical) solution.

11. Solve the following LP problems graphically.

$$\text{a)} \begin{cases} \max & 5x_1 - 3x_2 \\ \text{s. t.} & x_1 + 2x_2 \leqslant 4 \\ & x_1 + 3x_2 \geqslant 6 \\ & x_1, x_2 \geqslant 0 \end{cases} \qquad \text{b)} \begin{cases} \max & x_1 + 3x_2 \\ & -x_1 - x_2 \leqslant -3 \\ & -x_1 + x_2 \leqslant -1 \\ & x_1 + 2x_2 \leqslant 4 \\ & x_1, x_2 \geqslant 0 \end{cases}$$