Homework 3.3
8 points. End term: 12-th week (december 18, 2025)

1. (3 points) Players **A** and **B** simultaneously call out one of the numbers one, two, three or four. The sum of the two numbers is won by player **A** if the sum is an even number not divisible by three, otherwise the sum is won by player **B**.

(a) *(1 point)* Set up the matrix of payoffs.

(b) *(2 points)* Solve the game and find its value: try first by eliminating weak dominated strategies, then look for a saddle point (maximin/minimax), and, only after that, try with mixed strategies (in the latter case, just formulate the corresponding linear problems and find the mixed strategies).

2. (5 points) Four political parties P_1, P_2, P_3, and P_4 have 30, 20, 25, and 25 representatives respectively, in a parliament of 100 representatives. To pass a \$10 billion spending bill, at least 50 votes are needed, to pass a \$20 billion spending bill, at least 75 votes are needed, and to pass a \$30 billion spending bill, at least 90 votes are needed.

After passing a bill the parties who supported it will decide how the money should be allocated. If no bill is passed, then everyone gets nothing to allocate. The parties will try to pass the larger possible bill, by cooperating with each other. Use this context for modelling a cooperative game with transferable utility.

(a) *(1 point)* Find the characteristic function of the game.

(b) *(1.5 points)* Describe the core of the game as a set of linear inequalities.

(c) *(2.5 points)* Compute the Shapley value for this game.