Homework 2.3

9 points. End term: 7-th week (november 13, 2025)

1. (1 **point)** Solve one of the following problems using the Dual Simplex method.

$$
\textbf{(a)}\begin{cases}
\min & z = 2x_1 + x_3 + 3x_4 \\
\text{s. t.} & 2x_1 + 3x_2 + x_3 - x_4 \geqslant 6 \\
 & -x_1 + 2x_2 - x_3 + x_4 \leqslant -4 \\
 & x_1 + x_2 + 2x_3 \geqslant 2 \\
 & x_1, x_2, x_3, x_4 \geqslant 0
\end{cases}
\qquad
\textbf{(b)}\begin{cases}
\min & z = x_1 + 2x_2 + 3x_3 + x_4 \\
\text{s. t.} & 2x_1 + 2x_2 + x_3 + x_4 \geqslant 2 \\
 & -x_1 + 2x_2 + x_3 - 2x_4 \leqslant 6 \\
 & x_1 + 2x_2 + 2x_3 + 3x_4 \geqslant 4 \\
 & x_1, x_2, x_3, x_4 \geqslant 0
\end{cases}
$$

2. (2 **points)** The following tableau corresponds to an iteration of the Dual Simplex method (using Bland's rule) when solving a minimization LP problem:

	x_1	x_2	x_3	x_4	x_5	x_6	x_7	RHS
	0	a_3	-1	1	2	a_9	0	-2
	a_1	1	0	a_5	-1	0	0	1
	0	a_4	1	a_6	a_7	0	a_{10}	a_{12}
z	a_2	1	2	0	a_8	1	a_{11}	4

(i) What can you say about the parameters a_i, $1 \leqslant i \leqslant 12$?

(ii) What are the basic and non-basic variables? Provide the current values of all the variables.

Find constraints on the parameters a_i, $1 \leqslant i \leqslant 12$ such that the following statements are true.

(ii) The current basis is primal infeasible.

(iii) The current basis is primal infeasible x_5 enters the basis, and the resulted basis is still primal infeasible.

(iv) The current basis is primal infeasible and x_3 enters the basis.

(v) The current basis is primal infeasible, x_2 enters the basis, and the change in the objective is zero.

(vi) The problem is infeasible.

3. (5 **points)** Devise an implementation (C, C++, C#, Java) for the Dual Simplex algorithm (with Bland rule: breaking ties by choosing the smallest index variable) using the following pseudo-code (the simplex tableau has $(m+1) \times (n+1)$ dimensions) :

// if $t_{i,n+1} \geqslant 0$, $\forall 1 \leqslant i \leqslant m$, the dual simplex table is optimal: the basic current solution is optimal.

while $(\exists\, t_{i,n+1} < 0,\ 1 \leqslant i \leqslant m)$ **{**

  let $1 \leqslant k \leqslant m$ **such that** $t_{k,n+1} < 0$;

  // k is the pivot row, i.e., on row k will be the leaving basic variable.

  if $(t_{k,j} \geqslant 0,\ \forall\, 1 \leqslant j \leqslant n)$ *// the problem is infeasible.*

    return;

  let $1 \leqslant l \leqslant n$ **such that** $\left|\dfrac{t_{m+1,l}}{t_{k,l}}\right| = \min\left\{\left|\dfrac{t_{m+1,j}}{t_{k,j}}\right| \ :\ 1 \leqslant j \leqslant n, t_{k,j} < 0\right\}$

  // l is the pivot column, on column l will be the entering basic variable

  for $i = \overline{1, m+1}$, $i \neq k$

$\quad$ **for** $j = \overline{1, n+1}$, $j \neq l$

$\quad\quad t_{i,j} \leftarrow \dfrac{t_{i,j} t_{k,l} - t_{i,l} t_{k,j}}{t_{k,l}}$; // *pivoting rule*.

for $i = \overline{1, m+1}$, $i \neq k$

$\quad t_{i,l} \leftarrow 0$;

for $j = \overline{1, n+1}$, $j \neq l$

$\quad t_{k,j} \leftarrow \dfrac{t_{k,j}}{t_{k,l}}$;

$t_{k,l} \leftarrow 1$;

}

4. (1 **point**) Run the above implemented algorithm on the problems **1 (a)** and **(b)** and check your results using **Gurobi Solver**.