Homework 1.3

11 points. End term: 4-th week (october 23, 2025)

1. ($2 \times 1 =$ **2 points**) Solve the following two LP problems using the simplex algorithm:

(a)
$$\begin{cases} \min & z = 5x_1 - 2x_2 - 4x_3 - x_4 \\ \text{s. t.} & 4x_1 + 3x_2 + 2x_3 + x_4 \leqslant 8 \\ & x_1 + 2x_3 + x_4 \leqslant 5 \\ & 2x_1 + 3x_2 + x_3 + 2x_4 \leqslant 7 \\ & x_1, x_2, x_3, x_4 \geqslant 0 \end{cases}$$

(b)
$$\begin{cases} \min & z = -6x_1 + x_2 - 2x_3 \\ \text{s. t.} & x_1 + 2x_2 + x_3 \leqslant 6 \\ & 2x_1 + x_2 + 3x_3 \leqslant 8 \\ & 2x_1 + 3x_2 + 4x_3 \leqslant 10 \\ & x_1, x_2, x_3 \geqslant 0 \end{cases}$$

2. (2 points) The following tableau corresponds to an iteration of the simplex method (use Bland's rule):

	x_1	x_2	x_3	x_4	x_5	x_6	RHS
	a_1	-1	1	0	2	1	a_{11}
	0	a_3	0	a_5	0	1	2
	1	-2	0	a_6	a_8	3	a_{12}
z	a_2	a_4	0	a_7	a_9	a_{10}	1

(i) What can you say about the parameters a_i, $1 \leqslant i \leqslant 12$?

(ii) What are the basic and non-basic variables? Provide the current values of all the variables.

Find constraints on the parameters $a_1, a_2, \ldots, a_{12}$ such that the following statements are true.

(iii) The current basis is optimal.

(iv) The current basis is the unique optimal basis.

(v) The current basis is optimal but alternative optimal bases exists.

(vi) The problem is unbounded.

(vii) x_6 enters the basis and the change in the objective is zero.

(viii) x_5 enters the basis.

3. (6 points) Devise an implemetation (C, C++, C#, Java) for the simplex algorithm (use Bland's rule) using the following pseudocode (the simplex tableau has $(m+1)\times(n+1)$ dimensions):

// if $t_{m+1,k} \geqslant 0$, $\forall k$, the simplex table is __optimal__: the current basic feasible solution is optimal.

while $(\exists\, t_{m+1,j} < 0,\ j \leqslant n)$ **{**

let $1 \leqslant l \leqslant n$ **such that** $t_{m+1,l} < 0$**;**

// l is the pivot column, i.e., on column l will be the __entering basic variable__

if $(t_{h,l} \leqslant 0,\ \forall\, 1 \leqslant h \leqslant m)$

// the __problem__ has no optimum solution (although it has feasible solutions), i.e. the objective functions is __unbounded__

return;

let $1 \leqslant k \leqslant m$ **such that** $\dfrac{t_{k,n+1}}{t_{k,l}} = \min\left\{\dfrac{t_{h,n+1}}{t_{h,l}} \ :\ t_{h,l} > 0\right\}$

// k is the pivot row, on line k will be the ***leaving basic variable***

for $i = \overline{1, m+1}$**,** $i \neq k$

  for $j = \overline{1, n+1}$**,** $j \neq l$

    $t_{i,j} \leftarrow \dfrac{t_{i,j}t_{k,l} - t_{i,l}t_{k,j}}{t_{k,l}}$**;** *//* ***pivoting rule***

for $i = \overline{1, m+1}$**,** $i \neq k$

  $t_{i,l} \leftarrow 0$**;**

for $j = \overline{1, n+1}$**,** $j \neq l$

  $t_{k,j} \leftarrow \dfrac{t_{k,j}}{t_{k,l}}$**;**

$t_{k,l} \leftarrow 1$**;**

}

4. (1 point) Run the above implemented algorithm on the problems **1 (a)** and **(b)** and check your results using **Gurobi Solver**.

Bland's rule:

- among all candidates for entering column, choose the one with the smallest index, i.e. replace

  let $1 \leqslant l \leqslant n$ **such that** $t_{m+1,l} < 0$**;**

  with

  let $1 \leqslant l \leqslant n$ **such that** $l = \min\{j \, : \, t_{m+1,j} < 0\}$**;**

- among all rows for which the minimum ratio $\min\left\{\dfrac{t_{h,n+1}}{t_{h,l}} \, : \, t_{h,l} > 0\right\}$ is the same, choose the row corresponding to the basic variable with the smallest index.