Homework 1.2

8 points. End term: 4-th week (october 23, 2025)

1. (2 points) Let $\mathbf{x}$ be a feasible point for the constraints $\{\mathbf{Ax} = \mathbf{b}, \mathbf{x} \geqslant \mathbf{0}\}$ that is not an extreme point. Prove that there exists a vector $\mathbf{y} \neq 0$, with the following properties: $\mathbf{Ay} = \mathbf{0}$, and $x_i = 0 \Rightarrow y_i = 0$.

2. (3 points) True or false (give an example or a short proof).

(a) *(1 point)* There exist LP problems for which the set of optimal solutions forms an entire closed half-line.

(b) *(1 point)* There exist LP unbounded problems with bounded feasible region.

(c) *(1 point)* There exist LP problems that have two feasible solutions, $\mathbf{x}^1, \mathbf{x}^2$, such that any $\mathbf{x} \in (\mathbf{x}^1, \mathbf{x}^2]$ is an optimal solution, but $\mathbf{x}^1$ is not.

3. (3 points) For the following two LP problems, write the systems in standard form, determine all the basic solutions (feasible and infeasible), and find optimal basic solutions (if any).

(a) (1.5 points) $\begin{cases} \max & z = 4x_1 + 3x_2 \\ \text{s. t.} & 3x_1 + x_2 \geqslant 9 \\ & 2x_1 + 4x_2 \leqslant 16 \\ & x_1, x_2 \geqslant 0 \end{cases}$

(b) (1.5 points) $\begin{cases} \max & z = 2x_1 + 3x_2 \\ \text{s. t.} & x_1 + 4x_2 \leqslant 8 \\ & x_1 + 3x_2 \leqslant 6 \\ & 2x_1 + x_2 \leqslant 4 \\ & x_1, x_2 \geqslant 0 \end{cases}$