

Homework 1.1

6 points. End term: 4-th week (october 23, 2025)

1. (1 point) Let $G = (V, E)$ be a graph (having no isolated vertices) with non-negative weights on its edges, $w : E \rightarrow \mathbb{Z}_+$. The w -coloring problem is: find a coloring of the vertices of G , $c : V \rightarrow \mathbb{N}$, with minimum number of colors such that

$$|c(u) - c(v)| \geq w(uv), \forall uv \in E.$$

Model this problem as an ILP problem.

2. (1 point) Let $G = (V, E)$ be a graph with non-negative weights on its edges, $w : E \rightarrow \mathbb{R}_+$. The w -matching problem asks to find a maximum weight matching in G . Model this problem as an ILP problem.

3. (2 points) EU wants to plan its electricity capacity for the next k years. The EU Energy department forecasts the demand (in MW) to be d_i in year $i \in \{1, 2, \dots, k\}$. The existing capacity (most of it being carbon-intensive energy) at the beginning of year i is c_i (MW). There are two legal ways for expanding electric capacity: renewable energy and nuclear energy. The capital cost per MW of renewable energy becoming available at the beginning of the i th year is e_i and the same cost for nuclear energy is n_i . For safety reasons it was decided that the nuclear energy should not be more than 25% in the total energy mix.

- (a) Model as an LP problem the following requirement: find the least expensive energy expansion plan.
- (b) Review your solution if we know that a nuclear energy plant should last no more than m years.

4. (2 points) Solve using the geometric approach the following two linear programs.

$$(a) \text{ (1 point)} \left\{ \begin{array}{l} \max \quad z = 2x_1 + 3x_2 \\ \text{s. t.} \quad x_1 + x_2 \geq 4 \\ \quad \quad x_1 + 2x_2 \geq 6 \\ \quad \quad x_1 - 2x_2 \leq 4 \\ \quad \quad 4x_1 - x_2 \geq 8 \\ \quad \quad x_1, x_2 \geq 0 \end{array} \right. \quad (b) \text{ (1 point)} \left\{ \begin{array}{l} \max \quad z = 3x_1 + 2x_2 \\ \text{s. t.} \quad -x_1 + x_2 \leq 3 \\ \quad \quad 2x_1 + x_2 \leq 8 \\ \quad \quad x_1 + 3x_2 \geq 9 \\ \quad \quad x_1, x_2 \geq 0 \end{array} \right.$$